Prove that if n is a positive integer and $x \geq -1$ then

$$(1+x)^n \geq 1 + nx.$$

Proof: The proof is by induction on n.

If n = 1 we have

$$(1 + x)^1 = 1 + x,$$

so the result is true.

Suppose then that $(1+x)^n \geq 1 + nx$. Then, since $(1 + x) \geq 0$, and by the induction hypotheseis:

$$\begin{aligned}(1+x)^{n+1} &= (1+x)(1+x)^n \\ &\geq (1+x)(1+nx)\end{aligned}.$$

Continuing,

$$\begin{aligned}&(1+x)(1+nx) \\ &= 1+(n+1)x+x^2 \\ &\geq 1+(n+1)x\end{aligned}$$

So

$$(1+x)^{n+1} \geq 1+(n+1)x,$$

which establishes the induction.