Let Q be the set of all rationals and F be the set of all functions from Q to Q. Prove that F is not countable.

Proof: Suppose F is countable. Then $F = \{f_i : i = 1, 2, 3, ..\}$, where f_i is a function from Q to Q, and any function f from Q to Q is an f_i, for some integer i.

Next, since Q is countable, $Q = \{r_j : j = 1, 2, 3, ..\}$.

So define the function f from Q to Q by

$$f(r_j) = f_j(r_j) + 1 \text{ for } j = 1, 2, 3, ..$$

Then $f(r_i) = f_i(r_i) + 1 \Rightarrow f \neq f_i$ for any i. This contradicts the statement that every f from Q to Q is an f_i for some i.