MATH 3B (Winter 2002)
Instructor: Roberto Schonmann
Midterm Exam

Last Name:

First and Middle Names: Solutions

Signature:

UCLA id number (if you are an extension student, say so):

Circle the discussion section in which you are enrolled:

2A (Tue. 9am, Madeleine) 2B (Thur. 9am, Madeleine)

2C (Tue. 9am, Brian) 2D (Thur. 9am, Brian)

2E (Tue. 9am, Suneel) 2F (Thur. 9am, Suneel)

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	6	Total
Score							

1) (10 points) Suppose that

$$f(x) = \begin{cases} x & \text{if } x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

Find a such that

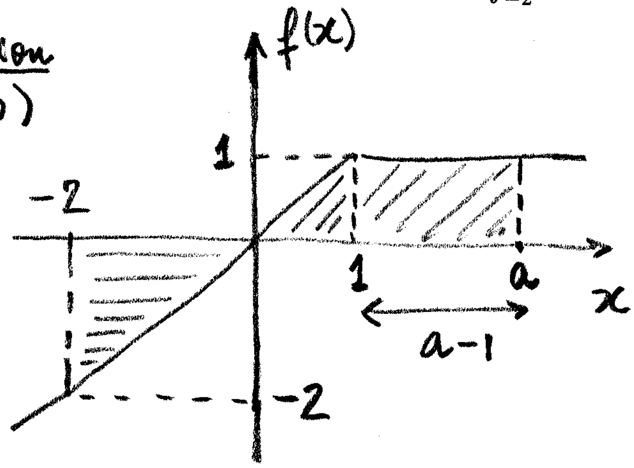
$$\int_{-2}^{a} f(x)dx = 0$$

First Solution:

For any function $\int_a^a f(x)\,dx = 0$

So $\boxed{a = -2}$ solves the problem

2nd solution (with $a>0$)



Equate areas

$$\frac{2\times 2}{2} = \frac{1\times 1}{2} + (a-1)\times 1$$

$$2 = \frac{1}{2} + a - 1$$

$$a = 2 - \frac{1}{2} + 1 = \boxed{\frac{5}{2}}$$

2) (10 points) Compute the following indefinite integral:

$$\int \left(\sqrt{3x} + \sqrt{e^{5x}}\right) dx$$

$$\int \sqrt{3}\, x^{1/2}\, dx + \int e^{5/2\, x}\, dx = \sqrt{3} \cdot \frac{2}{3} x^{3/2} + \frac{2}{5} e^{\frac{5}{2}x}$$

$$= \frac{2}{\sqrt{3}} \sqrt{x^3} + \frac{2}{5} \sqrt{e^{5x}} + C$$

3) (10 points) Compute the average value of the function $h(x) = 3^x$ on the interval $[-1, 2]$.

$$f_{avg} = \frac{1}{2-(-1)} \int_{-1}^{2} e^{(\ln 3)x}\, dx$$

$$= \frac{1}{3}\left[\frac{e^{(\ln 3)x}}{\ln 3}\right]_{-1}^{2} = \frac{1}{3\ln 3}\left[3^x\right]_{-1}^{2}$$

$$= \frac{1}{3\ln 3}\left(9 - \frac{1}{3}\right) = \boxed{\frac{26}{9\ln 3}}$$

4) (10 points) Compute the following indefinite integral:

$$\int \frac{(\ln x)^3}{x}\, dx$$

substitution

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\, dx$

$$\int \frac{(\ln x)^3}{x}\, dx = \int u^3\, du$$

$$= \frac{u^4}{4} = \boxed{\frac{(\ln x)^4}{4} + C}$$

5) (10 points) Compute the following indefinite integral:

$$\int \sqrt{x}\ln x\,dx$$

By parts

$$\int \underbrace{x^{1/2}}_{f'(x)} \underbrace{\ln x}_{g(x)} = \frac{2}{3}x^{3/2}\ln x - \int \frac{2}{3}x^{3/2}\cdot\frac{1}{x}\,dx$$

$f(x) = \frac{2}{3}x^{3/2}$

$g'(x) = \frac{1}{x}$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{1/2}\,dx$$

$$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\cdot\frac{2}{3}x^{3/2}$$

$$= \frac{2}{3}x^{3/2}\left(\ln x - \frac{2}{3}\right) + C$$

6) (10 points) Compute

$$\int_0^\infty \frac{6}{5+5x^2}\,dx$$

$$\int_0^\infty \frac{6}{5+5x^2}\,dx = \frac{6}{5}\lim_{b\to\infty}\int_0^b \frac{1}{1+x^2}\,dx = \frac{6}{5}\lim_{b\to\infty}\Big[\arctan(x)\Big]_0^b$$

$$= \frac{6}{5}\lim_{b\to\infty}\Big(\arctan(b) - \arctan(0)\Big) = \frac{6}{5}\cdot\frac{\pi}{2} = \boxed{\frac{3\pi}{5}}$$