MATH 3A (Fall 2001)
Instructor: Roberto Schonmann
Midterm Exam

Last Name: Solutions

First and Middle Names:

Signature:

UCLA id number (if you are an extension student, say so):

Circle the discussion section in which you are enrolled:

2A (Tue. 9am, Rahul) 2B (Thur. 9am, Rahul) 2C (Tue. 9am, Tim)

2D (Thur. 9am, Tim) 2E (Tue. 9am, Brian) 2F (Thur. 9am, Brian)

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam.

Good Luck !

Question	1	2	3	4	5	6	7	8	Total
Score									

Find the limits below

1) (10 points) $\lim_{x\to 0} \frac{x^3+2x-1}{\frac{\sin x}{x}+15} = \frac{-1}{1+15} = -\frac{1}{16}$

(Used limit laws and $\lim_{x\to 0} \frac{\sin x}{x} = 1$)

2) (10 points) $\lim_{x\to -\infty} \frac{x^5-x^3+4x^2+39}{9x^2-6} = \lim_{x\to -\infty} \frac{x^5}{9x^2} \frac{1-\frac{1}{x^2}+\frac{4}{x^3}+\frac{39}{x^5}}{1-\frac{6}{9x^2}}$

$= \left(\lim_{x\to -\infty} \frac{x^3}{9}\right) \cdot 1 = -\infty$

3) (10 points) Compute b so that $f(x)$ below becomes continuous at $x = 1$.

$$f(x) = \begin{cases} \frac{3x^2-9x+6}{x-1} & \text{if } x < 1 \\ \frac{2}{b} + \cos(x-1) & \text{if } x \geq 1 \end{cases}$$

Need $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

Factor $3x^2 - 9x + 6$: roots: $\frac{9 \pm \sqrt{81-72}}{6} = \frac{9 \pm 3}{6} = \begin{matrix} \nearrow 2 \\ \searrow 1 \end{matrix}$

$$3x^2 - 9x + 6 = 3(x-2)(x-1)$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{3(x-2)(x-1)}{(x-1)} = \lim_{x \to 1^-} (3(x-2)) = -3$$

$$\lim_{x \to 1^+} \frac{2}{b} + \cos(x-1) = \frac{2}{b} + 1$$

$$-3 = \frac{2}{b} + 1 \therefore \frac{2}{b} = -4 \therefore b = -\frac{2}{4} = \boxed{-\frac{1}{2}}$$

4) (10 points) Find the equation of the tangent line to the graph of the function $f(x) = x + \sqrt{x}$, through the point $(4, 6)$.

$$f(x) = x + x^{1/2} \quad \therefore \quad f'(x) = 1 + \frac{1}{2} x^{-1/2} = 1 + \frac{1}{2\sqrt{x}}$$

$$f'(4) = 1 + \frac{1}{2 \cdot 2} = \frac{5}{4}$$

tangent line: $y - 6 = (x-4) \cdot \frac{5}{4}$

$$\therefore y - 6 = \frac{5x}{4} - 5$$

$$\therefore y = \frac{5x}{4} + 1$$

5) (10 points) Compute the first and second derivatives of the function $f(x) = \sin(x^2+1)$.

$$f'(x) = \cos(x^2+1) \cdot \frac{d}{dx}(x^2+1) = 2x\cos(x^2+1)$$

$$f''(x) = 2\cos(x^2+1) + 2x \cdot \left(-\sin(x^2+1) \cdot \frac{d}{dx}(x^2+1)\right)$$

$$= 2\cos(x^2+1) - 2x\sin(x^2+1) \cdot 2x$$

$$= 2\cos(x^2+1) - 4x^2\sin(x^2+1)$$

6) (10 points) Compute the derivative of the function $h(x) = \frac{3^x}{x}$.

$$h(x) = \exp((\ln 3) \cdot x) \,/\, x$$

$$h'(x) = \frac{\exp((\ln 3)x) \cdot \frac{d}{dx}((\ln 3) \cdot x) \cdot x - \exp((\ln 3)x) \cdot 1}{x^2}$$

$$= \frac{\exp((\ln 3)x) \cdot \ln(3) \cdot x - \exp((\ln 3)x)}{x^2}$$

$$= \frac{(\ln 3) \cdot x \cdot 3^x - 3^x}{x^2}$$

7) (10 points) Compute the derivative of the function $g(x) = \ln \ln x$.

$$g'(x) = \frac{1}{\ln x} \frac{d}{dx} \ln x = \frac{1}{x \ln x}$$

8) (10 points) Use a linear approximation to the function $f(x) = \exp(x)$ at $x = 0$ to obtain an approximate value for $\exp(0.01)$.

$$f'(x) = \exp(x)$$

$$L(x) = f(0) + f'(0) \cdot (x-0) = 1 + 1 \cdot (x-0) = 1+x$$

$$\exp(0.01) \approx L(0.01) = 1.01$$