MATH 3A (Winter 2002)
Instructor: Roberto Schonmann
Midterm Exam

Last Name:

First and Middle Names: Solutions

Signature:

UCLA id number (if you are an extension student, say so):

Circle the discussion section in which you are enrolled:

2A (Tue. 8am, Stephanie) 2B (Thur. 8am, Stephanie)

2C (Tue. 8am, Kathleen) 2D (Thur. 8am, Kathleen)

2E (Thur. 8am, Sumni)

- Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	6	7	Total
Score								

Find the limits below

1) (10 points) $\lim_{x\to 0} \frac{\sin x}{\sqrt{x}}$

$$\lim_{x\to 0} \frac{\sin x}{\sqrt{x}} = \lim_{x\to 0} \left(\frac{\sqrt{x}}{\sqrt{x}} \frac{\sin x}{\sqrt{x}} \right)$$

$$= \lim_{x\to 0} \left(\sqrt{x} \frac{\sin x}{x} \right) = \left(\lim_{x\to 0} \sqrt{x} \right) \cdot \left(\lim_{x\to 0} \frac{\sin x}{x} \right)$$

$$= 0 \cdot 1 = \boxed{0}$$

2) (10 points) $\lim_{x\to\infty} \frac{e^{3x}}{2+e^{3x}}$

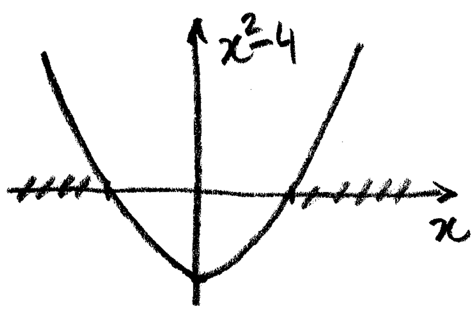
$$\lim_{x\to\infty} \frac{e^{3x}}{2+e^{3x}} = \lim_{x\to\infty} \frac{e^{3x}}{e^{3x}} \frac{1}{2/e^{3x}+1}$$

$$= \lim_{x\to\infty} \frac{1}{2/e^{3x}+1} = \frac{1}{1} = \boxed{1}$$

3) (10 points) For what values of x is the function $f(x) = \sqrt{x^2 - 4}$ continuous?

Polynomials, square roots are continuous wherever defined. Composition of continuous functions is continuous. So $f(x)$ is continuous wherever it is defined.

Need $x^2 - 4 \geq 0 \iff \underline{x \leq -2 \text{ or } x \geq 2}$



4) (10 points) Find the equation of the tangent line to the graph of the function $f(x) = x - \frac{1}{x}$, through the point $(1, 0)$.

$$f'(x) = 1 - \left(-\frac{1}{x^2}\right) = 1 + \frac{1}{x^2}$$

$$m = f'(1) = 1 + \frac{1}{1^2} = 2 \quad \text{(slope of tangent line)}$$

eq. of tangent line :

$$(y - 0) = m \cdot (x - 1) = 2(x - 1)$$

$$\therefore \boxed{y = 2x - 2}$$

5) (10 points) Suppose that $f'(x) = \frac{1}{x}$. Find $\frac{d}{dx} f(x^3 - 7)$.

Chain rule:

$$\frac{d}{dx} f(x^3-7) = f'(x^3-7) \cdot \frac{d}{dx}(x^3-7)$$

$$= \frac{1}{x^3-7} (3x^2) = \boxed{\frac{3x^2}{x^3-7}}$$

6) (10 points) Compute the second derivative of the function $h(x) = \cos(x^2)$.

$$h'(x) = -\sin(x^2) \cdot \frac{d}{dx} x^2 = -2x\sin(x^2)$$

$$h''(x) = -2\sin(x^2) - 2x\left(\cos(x^2) \cdot 2x\right)$$

$$= -2\sin(x^2) - 4x^2\cos(x^2)$$

7) (10 points) The position at time time t of a particle that moves along a line is given by a function $s(t)$. The derivative of $s(t)$ is the velocity $v(t)$, and the derivative of the velocity is the acceleration $a(t)$. Given that $s(t) = \sqrt{t+1}$, $t \geq 0$, find $a(3)$, that is, the acceleration at time $t = 3$.

$$s(t) = \sqrt{t+1}$$

$$s'(t) = \frac{1}{2\sqrt{t+1}} \cdot \frac{d}{dt}(t+1) = \frac{1}{2\sqrt{t+1}} = \frac{1}{2}(t+1)^{-1/2}$$

$$s''(t) = \frac{1}{2} \times \left(-\frac{1}{2}\right)(t+1)^{-3/2} = -\frac{1}{4(\sqrt[2]{t+1})^3}$$

$$s''(3) = -\frac{1}{4 \times (\sqrt[2]{4})^3} = -\frac{1}{4 \times 2^3} = \boxed{-\frac{1}{32}}$$