MATH 33A (Lecture 2, Fall 2003)
Instructor: Roberto Schonmann
Midterm Exam

Last Name:

First and Middle Names: Solutions

Signature:

UCLA id number (if you are an extension student, say so):

Circle the discussion section in which you are enrolled:

2A (Tue. 10am, Stephen) 2B (Thur. 10am, Stephen)

2C (Tue. 10am, Brian) 2D (Thur. 10am, Brian)

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. In questions where there is a "yes or no" answer, the grading is always based on the explanation rather than on the answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	6	7	Total
Score								

1) (10 points) What is the rank of the matrix below if a, b and c are different from 0?

$$\begin{bmatrix} a & a & b \\ 0 & 0 & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\begin{bmatrix} a & a & b \\ 0 & 0 & 0 \\ 0 & 0 & c \end{bmatrix} \div a$$

$$\begin{bmatrix} 1 & 1 & b/a \\ 0 & 0 & 0 \\ 0 & 0 & c \end{bmatrix} \text{ swap}$$

$$\begin{bmatrix} 1 & 1 & b/a \\ 0 & 0 & c \\ 0 & 0 & 0 \end{bmatrix} \div c$$

$$\begin{bmatrix} 1 & 1 & b/a \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad \triangle \text{ leading 1's}$$

There are 2 leading 1's in rref(A). Therefore

$$\boxed{\text{rank}(A) = 2}$$

2) (10 points) Do the vectors below form a basis of $\mathbb{R}^4$?

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 7 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 4 \\ 5 \\ 7 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

No. A basis of $\mathbb{R}^4$ must have exactly 4 vectors, and there are only 3 above.

3) (10 points) Find the inverse of the matrix below, or conclude that this matrix is not invertible.

$$\begin{bmatrix} 2 & 0 & 2 \\ 3 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 2 & 1 & 0 & 0 \\ 3 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \div 2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1/2 & 0 & 0 \\ 3 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ -3(\mathrm{I}) \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1/2 & 0 & 0 \\ 0 & 1 & -1 & -3/2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -(\mathrm{III}) \\ +(\mathrm{III}) \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & -1 \\ 0 & 1 & 0 & -3/2 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Answer $\begin{bmatrix} 1/2 & 0 & -1 \\ -3/2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

4) (10 points) If A is a square matrix and A^2 has all its entries 0, can you conclude that A has all its entries 0? Explain your answer (no credit will be given for right answer for wrong reason).

No. For instance, if $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

5) (10 points) Find the dimension of the kernel of the linear transformation T which corresponds to the matrix

$$A = \begin{bmatrix} 2 & 4 & 6 & -2 & 8 \\ 1 & 2 & 3 & -1 & 4 \end{bmatrix}$$

$A: 2 \times 5$ so $n=5$

$\dim(\ker(T)) = n - \operatorname{rank}(A) = 5 - \operatorname{rank}(A)$

Finding $\operatorname{rref}(A)$:

$$\begin{bmatrix} 2 & 4 & 6 & -2 & 8 \\ 1 & 2 & 3 & -1 & 4 \end{bmatrix} \div 2$$

$$\begin{bmatrix} 1 & 2 & 3 & -1 & 4 \\ 1 & 2 & 3 & -1 & 4 \end{bmatrix} -(I)$$

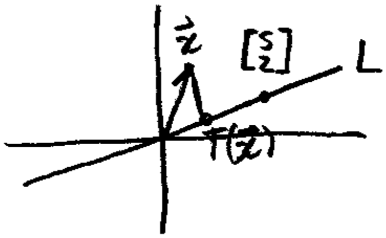
$$\begin{bmatrix} 1 & 2 & 3 & -1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \operatorname{rref}(A)$$

$\operatorname{rank}(A) = 1$

$$\boxed{\dim(\ker(T)) = 5 - 1 = 4}$$

6) (10 points) Find the image of the linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ which corresponds to the projection on

$$L = \left\{ k \begin{bmatrix} 5 \\ 2 \end{bmatrix} \; : \; k \in \mathbb{R} \right\}.$$



$$\boxed{\text{Im}\,(T) = L}$$

explanation: By definition $T(\vec{z})$ is always on L and any $\vec{y} \in L$ is $\vec{y} = T(\vec{y})$.

7) (10 points) If A is a 3×3 matrix, how many solutions can the equation $A\vec{x} = \vec{0}$ have? (Here $\vec{x} \in \mathbb{R}^3$ is the unknown.) For each answer to this question, give a corresponding example of A.

- If $\text{rank}(A) = 3$ then $A\vec{x} = \vec{0}$ has unique solution $\vec{x} = \vec{0}$ (since A is invertible). e.g. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- If $\text{rank}(A) < 3$ then there are infinitely many solutions. e.g.,

  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Then $A\vec{x} = \vec{0} \Leftrightarrow x_1 = 0$

  (x_2, x_3 arbitrary)