MATH 167 (Winter 2007)
Instructor: Roberto Schonmann
Midterm Exam

Last Name:

First and Middle Names: Solutions

Signature:

UCLA id number (if you are an extension student, say so):

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	Total
Score						

1) (10 points) The bimatrix below represents the payoffs in dollars to the two players of a game. As usual, the entry (x, y) indicates that player I receives x dollars and player II receives y dollars. Also, as usual, player I chooses a row and player II chooses a column, and each player prefers to receive more money than less money. Is this a strictly competitive game? Explain your answer.

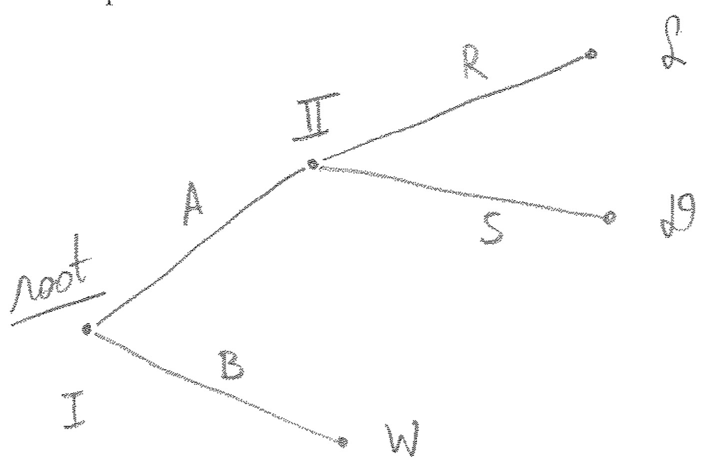
$$\begin{bmatrix} (\ 0, 0) & (1, -1) & (1, 2) \\ (-1, 6) & (0,\ \ 0) & (0, 1) \end{bmatrix}$$

Not a strictly competitive game,

because $(0,0) \prec_{\mathrm{I}} (1,2)$

and $(0,0) \prec_{\mathrm{II}} (1,2)$

2) (10 points) Find an example of a strictly competitive game of perfect information in which each one of the two players has exactly two pure strategies and there are exactly 2 Nash equilibria. You should draw the game tree, indicating clearly who plays at each node, the root of the tree, the outcomes at the terminal nodes, and the preferences over possible outcomes by the players. Label the edges of the tree and using these labels to denote the pure strategies, find the matrix of the game. Indicate clearly what the Nash equilibria are in the matrix.



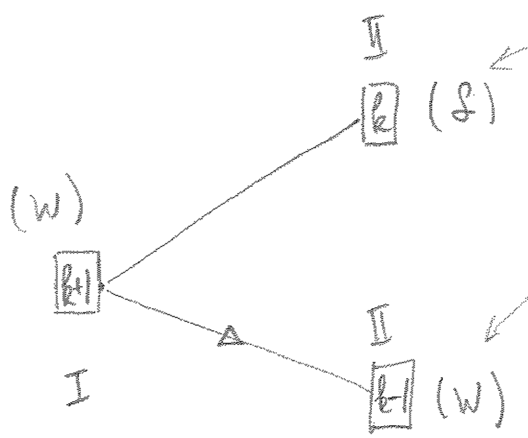
$\mathcal{L} <_I \mathcal{D} <_I W$

$W <_{II} \mathcal{D} <_{II} \mathcal{L}$

	R	S
A	$\mathcal{D}$	$\mathcal{L}$
B	W	W

Nash equilibria: BR, BS

3) (10 points) In a version of the game "pick-up-bricks" two players alternate taking 1 or 2 bricks from a pile that starts with n bricks. The players are called I and II, with I being the one who moves first. The winner is the player who removes the last brick. You are told what the value of this game is when $n = 1, 2, 3, ..., k-1, k$, for some k (for each n the value is $\mathcal{W}$, or $\mathcal{L}$, meaning that player I, or Player II, respectively, can assure a win in the game). If the value is $\mathcal{L}$ when $n = k-1$ and is $\mathcal{W}$ when $n = k$, what can you conclude the value to be when $n = k+1$?



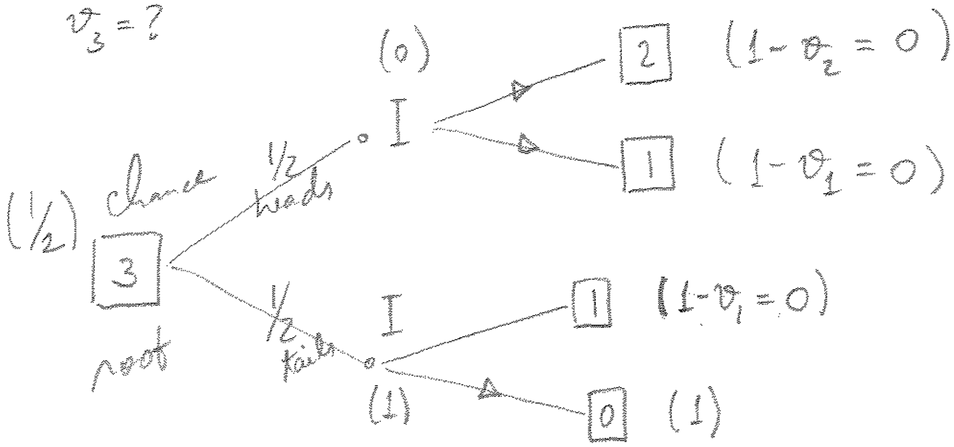
because player II moves at these points the values are as indicated, based on the information given above.

Answer : W

4) (10 points) The game "pick-up-bricks" from the previous problem is now modified in the following way. When it is a player's turn, a fair coin is tossed; if the coin shows heads, the player can remove 1 or 2 bricks; if the coin shows tails, the player can remove 2 or 3 bricks (unless there was only one brick left, in which case she can remove that single brick). As before, the winner is the player who removes the last brick. Suppose that the game starts with $n = 4$ bricks. What is the probability that player I wins the game when both players use the strategies that result from backwards induction?

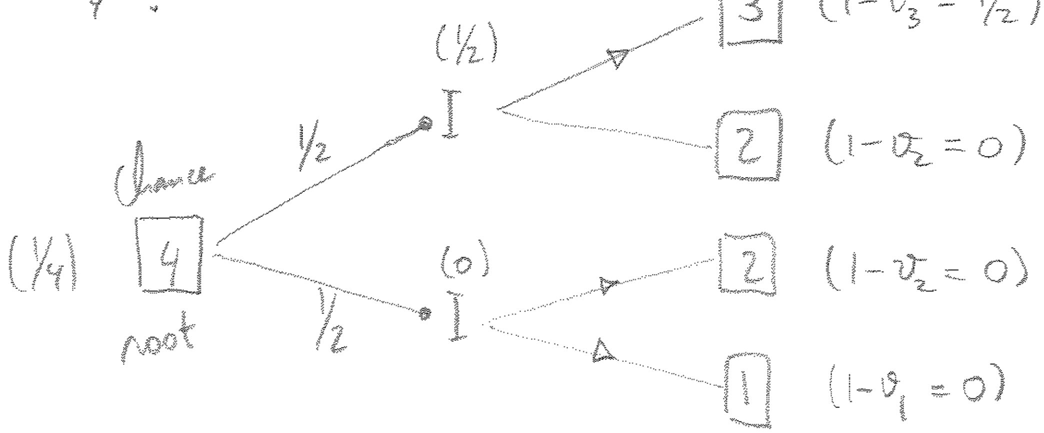
Clearly, with 1 or 2 bricks, player I wins for sure : $v_1 = v_2 = 1$

$v_3 = ?$



$\Rightarrow v_3 = 1/2$

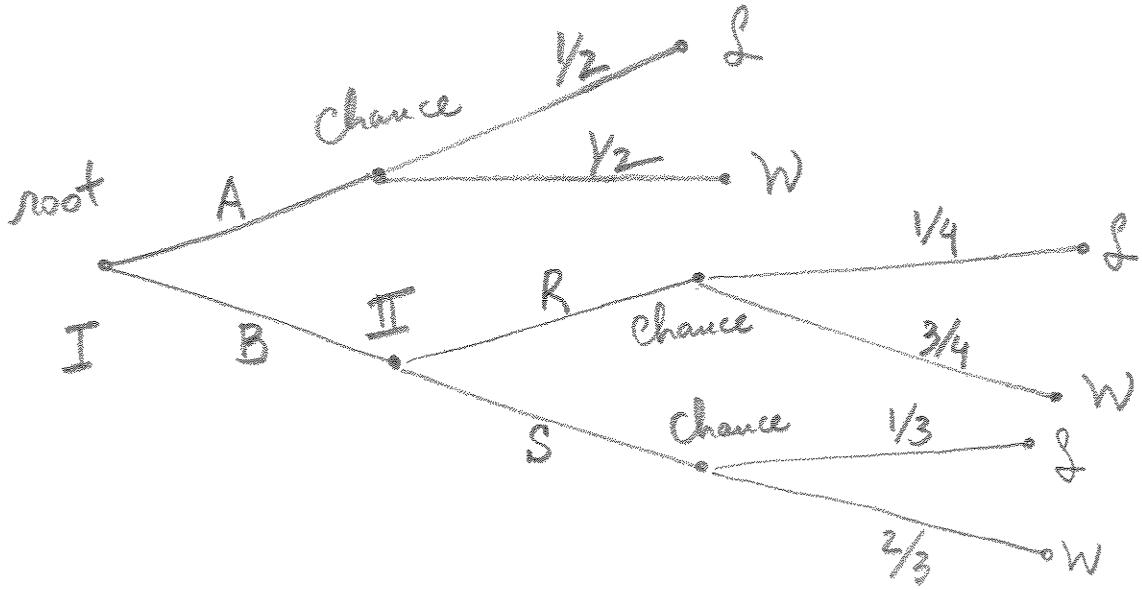
$v_4 = ?$



$\Rightarrow \underline{v_4 = \frac{1}{4}}$

Answer: $v_4 = 1/4$

5) (10 points) For the game of perfect information with chance moves given by the tree below, find its strategic form. In other words, find the matrix of this game in which the rows correspond to strategies of player I, the columns correspond to strategies of player II, and each entry is the probability that player I will win the game when the players use the strategies corresponding to that entry.



	R	S
A	1/2	1/2
B	3/4	2/3