MATH 167 (Winter 2007)
Instructor: Roberto Schonmann
Final Exam

Last Name:

First and Middle Names: Solutions

Signature:

UCLA id number (if you are an extension student, say so):

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	6	7	8	Total
Score									

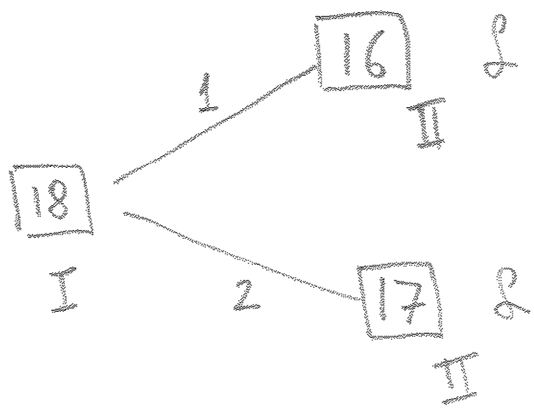
1) (10 points) The bimatrix below represents the payoffs in dollars to the two players of a game. As usual, the entry (x, y) indicates that player I receives x dollars and player II receives y dollars. Also, as usual, player I chooses a row and player II chooses a column, and each player prefers to receive more money than less money. Is this a strictly competitive game? Explain your answer.

$$\begin{bmatrix} (\ 0, 0) & (1, -1) & (0, 1) \\ (-1, 6) & (0,\ \ 0) & (0, 1) \end{bmatrix}$$

$(0,0) \sim_{I} (0,1)$ but $(0,0) \prec_{II} (0,1)$

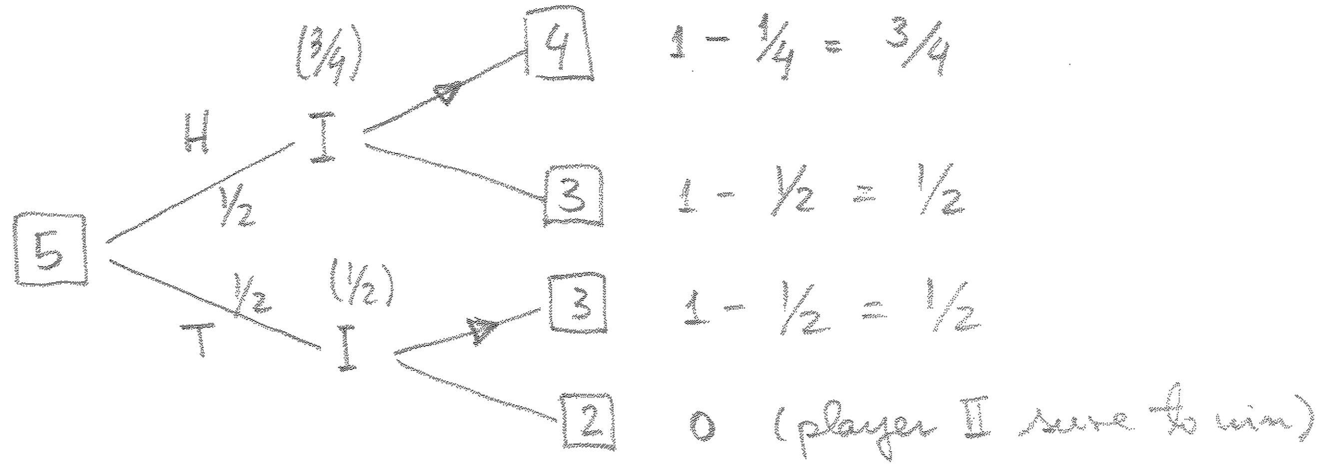
So not strictly competitive.

2) (10 points) In a version of the game "pick-up-bricks" two players alternate taking 1 or 2 bricks from a pile that starts with n bricks. The players are called I and II, with I being the one who moves first. The winner is the player who removes the last brick. Denote by v_n the value of the game (v_n is $\mathcal{W}$, or $\mathcal{L}$, meaning that player I, or Player II, respectively, can assure a win in the game). A friend told you that she has computed the value of the game for $n = 1, 2, ..., 20$. When you see that she found $v_{16} = \mathcal{W}$, $v_{17} = \mathcal{W}$, $v_{18} = \mathcal{W}$, you know that she made a mistake. Explain carefully why she must have made a mistake.

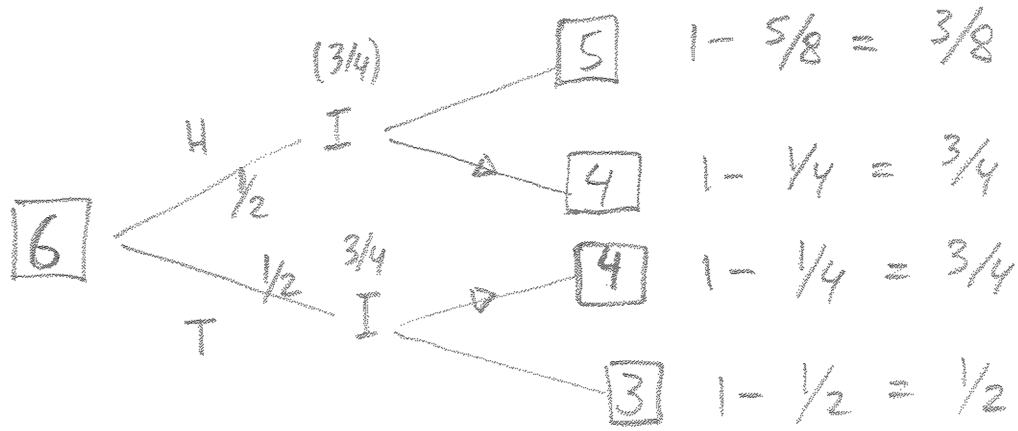


If $v_{16} = v_{17} = \mathcal{W}$, then using backwards induction as above gives $v_{18} = \mathcal{L}$. This shows that we cannot have $v_{16} = v_{17} = v_{18} = \mathcal{W}$.

3) (10 points) The game "pick-up-bricks" from the previous problem is now modified in the following way. When it is a player's turn, a fair coin is tossed; if the coin shows heads, the player can remove 1 or 2 bricks; if the coin shows tails, the player can remove 2 or 3 bricks (unless there was only one brick left, in which case she can remove that single brick). As before, the winner is the player who removes the last brick. Suppose that the game starts with n bricks. The value v_n of the game is the probability that player I wins the game when both players use the strategies that result from backwards induction. Suppose that you have already computed $v_3 = 1/2$ and $v_4 = 1/4$. Compute v_5 and v_6.



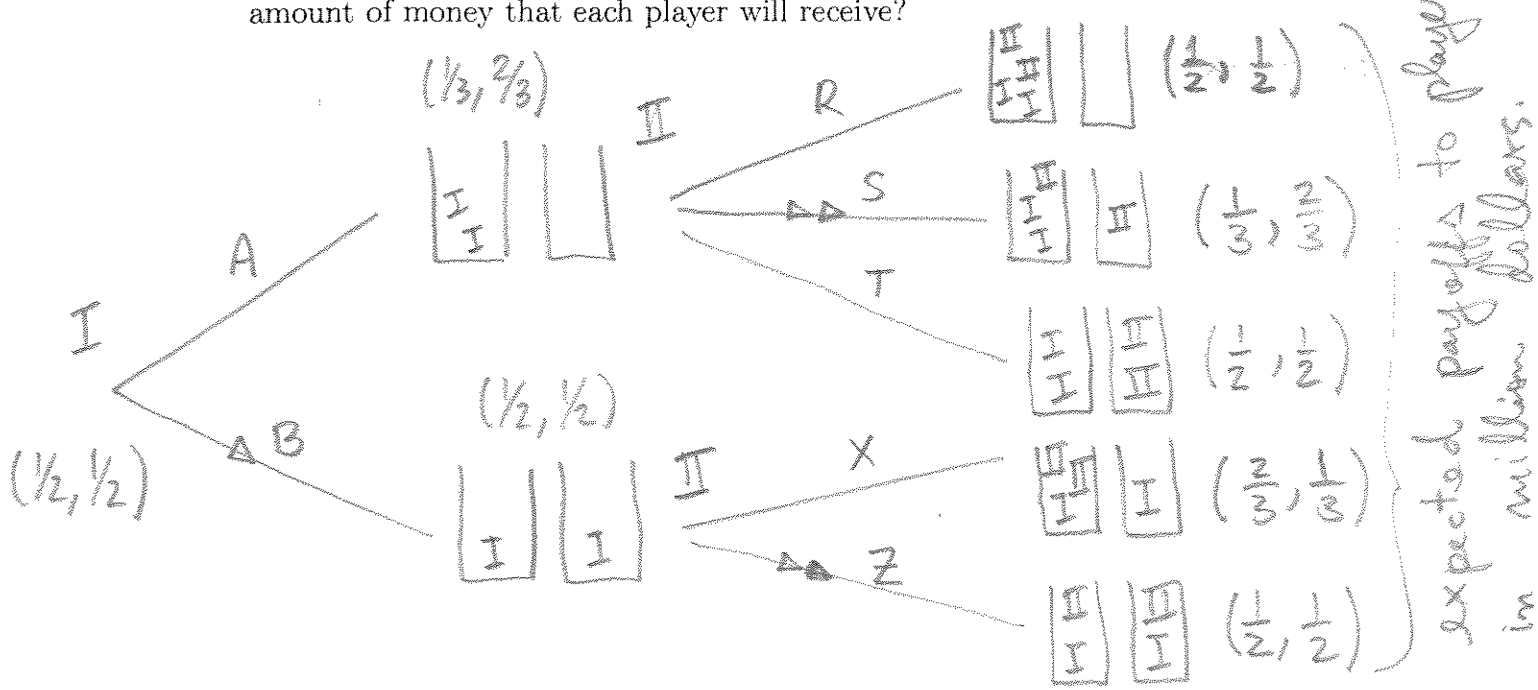
$$v_5 = \frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{2} = \frac{5}{8}$$



$$v_6 = \frac{1}{2} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} = \frac{3}{4}$$

4) (10 points) In this game player I has two chips marked with I and she moves first, placing them in any way she wants in two identical boxes. Player II also has two chips marked with II, and she moves after player I, placing her chips in either one of the two boxes (which are open with their content visible). A referee then selects a box at random and if this box is not empty she picks a chip at random from this box. The referee then pays the players according to the following rules. 1) If the box she picked at random was empty, each player receives $500,000. 2) If the box she picked at random was not empty, so that she picked a chip from it, player i receives $1,000,000 and the other player receives nothing, where i is the mark on the picked chip.

For the case in which the two players are risk neutral (and with the assumption of rationality), how will the game be played? What is the expected amount of money that each player will receive?



By backwards induction, they play BX SZ, meaning that player I puts the chips in different boxes, and then so does also player II. Each has expected payoff of .5 million dollars.

5) (10 points) Consider the game from the previous problem, but assume that the players are no longer supposed to be risk neutral. To assess the level of risk aversion of the players they are asked which lottery with only possible prizes being \$1,000,000 and \$0 each one of them finds as attractive as receiving \$500,000 for sure. Player I responds that it would be the one in which the probability of winning the \$1,000,000 is q_1. Player II responds that it would be the one in which the probability of winning the \$1,000,000 is q_2. Both players know this fact about the other. Represent the game in this case in bimatrix (strategic) form (note: player I will have two strategies and player II will have 6 strategies) and cross out the strategies that player II would eliminate by domination, regardless of the values of q_1 and q_2.

Tree diagram as in previous page

Utilities: $u_I(10^6)=1$, $u_I(0)=0$, $u_I(0.5\times10^6)=q_1$

$u_{II}(10^6)=1$, $u_{II}(0)=0$, $u_{II}(0.5\times10^6)=q_2$

	RX	RZ	SX	SZ	TX	TZ
A	$\frac{q_1}{2}+\frac{1}{4}, \frac{q_2}{2}+\frac{1}{4}$	$\frac{q_1}{2}+\frac{1}{4}, \frac{q_2}{2}+\frac{1}{4}$	$\frac{1}{3}, \frac{2}{3}$	$\frac{1}{3}, \frac{2}{3}$	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}$
B	$2/3, 1/3$	$1/2, 1/2$	$2/3, 1/3$	$1/2, 1/2$	$2/3, 1/3$	$1/2, 1/2$

RX: dominated by RZ; SX: dominated by SZ; TX: domin. by SZ; TZ: domin. by SZ

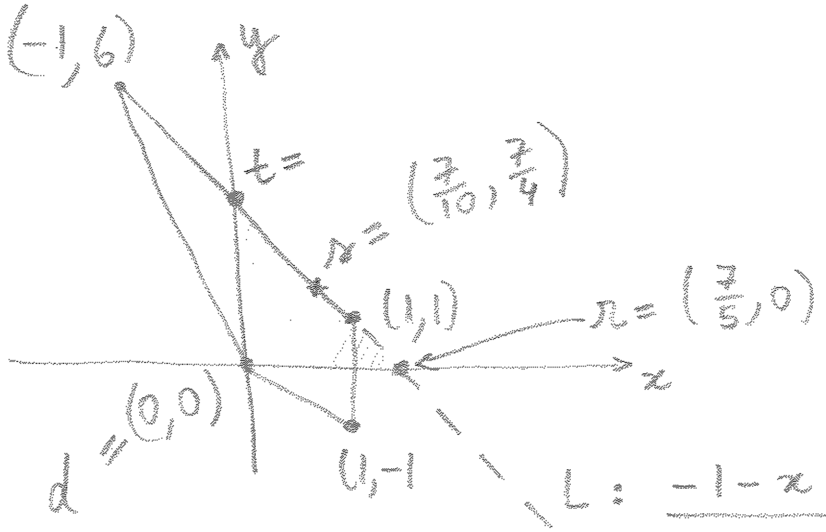
6) (10 points) Consider the following bimatrix game.

$$\begin{bmatrix} (\ 0,\ 0) & (1,-1) \\ (-1,6) & (1,\ \ 1) \end{bmatrix}$$

Compute the payoff to each player if they use Nash's standard bargaining solution (standard means that the bargaining powers are equal: $\alpha = \beta = 1/2$), with the understanding that the disagreement point is the only Nash equilibrium in pure strategies for this game.

	t_1	t_2
s_1	0, 0	1, −1
s_2	−1, 6	1, 1

$\Longrightarrow$ NE s_1, t_1 with payoff $(0,0)$



$L: \dfrac{-1-x}{-1-1} = \dfrac{6-y}{6-1} \quad \therefore \quad 5(x+1) = 2(6-y)$

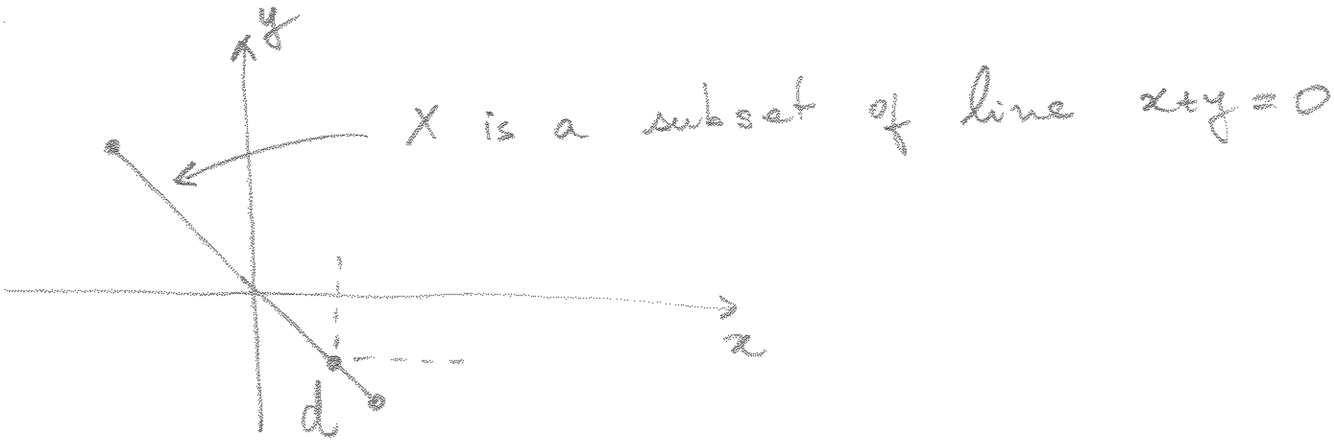
$\therefore \quad 5x + 2y = 7$

So $r = (7/5, 0)$, $t = (0, 7/2)$ (since both in L)

$s = \frac{1}{2} r + \frac{1}{2} t = \left(\frac{7}{10}, \frac{7}{4}\right)$ Belongs to bargaining set, since $0 \le \frac{7}{10} \le 1$

Answer: payoff to I $= 7/10$. Payoff to II $= 7/4$

7) (10 points) Suppose that two players want to use Nash's standard bargaining solution for a zero-sum game. How does the feasible set X look like? If the disagreement point is some point $d \in X$, what is the bargaining set? What is the solution in this case?



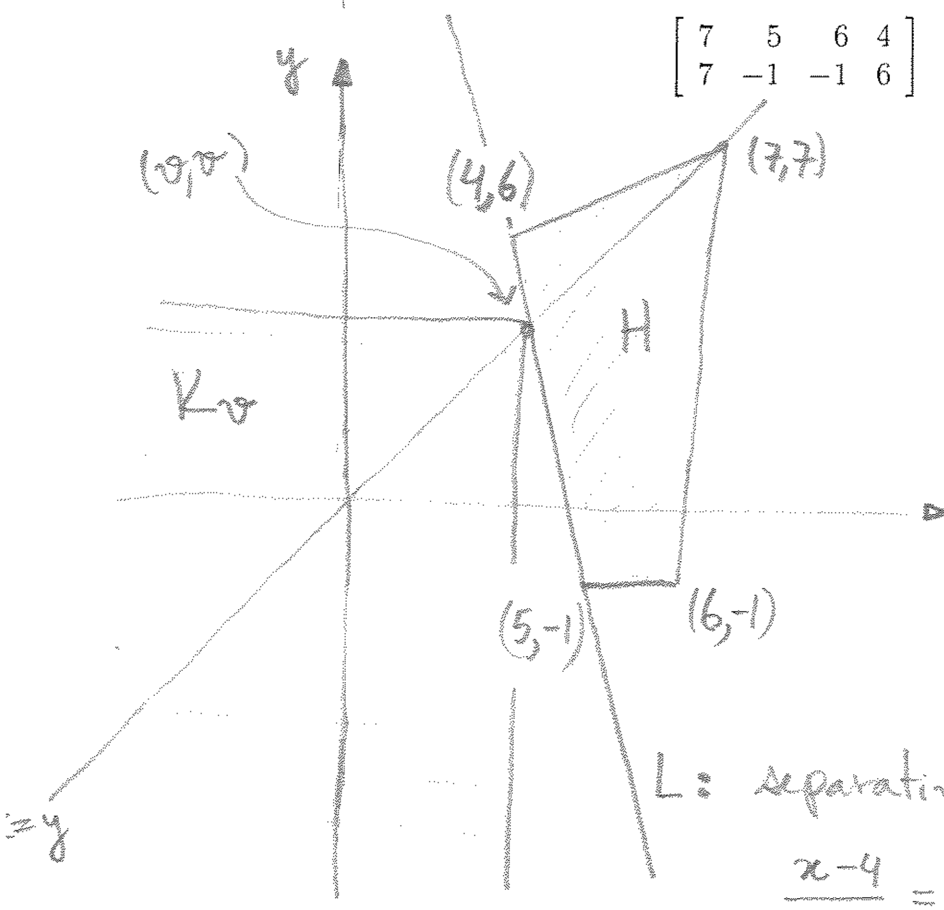
If $d \in X$, then bargaining set $= \{d\}$

(see picture)

So $F(X, d) = d$.

8) (10 points) Use the method of the separating line to compute the value of the matrix game below, as well as all security strategies for each player. Do not eliminate dominated strategies.

$$\begin{bmatrix} 7 & 5 & 6 & 4 \\ 7 & -1 & -1 & 6 \end{bmatrix}$$



L: separating line

$$\frac{x-4}{5-4} = \frac{y-6}{-1-6} \quad \therefore \quad 7x-28 = 6-y$$

$$\therefore \quad 7x+y = 34 \quad \therefore \quad \frac{7}{8}x + \frac{1}{8}y = \frac{17}{4}$$

$v = ?$ $\quad (v,v) \in L \Rightarrow 7v + v = 34 \Rightarrow v = \frac{17}{4}$

$\tilde{p} = ?$ $\quad L: \tilde{p}^T \begin{bmatrix} x \\ y \end{bmatrix} = v \Rightarrow \tilde{p} = \begin{bmatrix} 7/8 \\ 1/8 \end{bmatrix}$

$\tilde{q} = ?$ $\quad \tilde{q} = \begin{bmatrix} 0 \\ q_2 \\ 0 \\ q_4 \end{bmatrix}$ $\quad \begin{cases} q_2 \cdot 5 + q_4 \cdot 4 = 17/4 \\ q_2 \cdot (-1) + q_4 \cdot 6 = 17/4 \end{cases} \Rightarrow \tilde{q} = \begin{bmatrix} 0 \\ 1/4 \\ 0 \\ 3/4 \end{bmatrix}$