MATH 131A (Fall 2002, Lecture 2)
Instructor: Roberto Schonmann
Second Midterm Exam

Last Name:

First and Middle Names:

Signature:

UCLA id number (if you are an extension student, say so):

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	Total
Score						

1) (10 points) Define Cauchy sequence.

$\{a_n\}$ is Cauchy if

$\forall \varepsilon > 0 \quad \exists N$ s.t.

$$n, m \geq N \implies |a_n - a_m| \leq \varepsilon$$

2) (10 points) Show that if $\sup S = \infty$, then there is a sequence of points $s_n \in S$ such that $s_n \to +\infty$.

$\sup S = +\infty \implies S$ has no upper bound

$\implies \forall M \quad \exists s \in S \cap [M, \infty)$

Therefore one can take for each n

$s_n \in S \cap [n, \infty)$.

To show $s_n \to +\infty$:

$\forall M$ take $N \in \mathbb{N}$, $N \geq M$. Then

$n \geq N \implies s_n \geq n \geq N \geq M \qquad \square$

3) (10 points) Define the concept of limit points of a sequence $\{a_n\}$.

d is a limit point of $\{a_n\}$ if

$\forall \varepsilon > 0 \quad \forall N \quad \exists n \geq N$ s.t. $|a_n - d| \leq \varepsilon$.

4) (10 points) Provide an example of the following, or explain why this is not possible. A sequence $\{a_n\}$ with a limit point at 0 and with a subsequence $\{a_{n_k}\}$ such that $a_{n_k} \to \infty$.

Example:

$$a_n = \begin{cases} 0 & n \text{ even} \\ n & n \text{ odd} \end{cases}$$

Note that for $n_k = 2k-1$

$$a_{n_k} = a_{2k-1} = 2k-1 \to \infty.$$

5) (10 points) Suppose that f, g are continuous functions. Show that $f \cdot g$ is continuous. (You can use the definition of continuity and any property of sequences, but nothing else.)

Set $D = \mathrm{Dom}(f) \cap \mathrm{Dom}(g) = \mathrm{Dom}(f \cdot g)$

Suppose $x_n \to c$, $x_n \in D \ \forall n$.

Then $x_n \in \mathrm{Dom}(f)$ and hence $f(x_n) \to f(c)$, (I)

by the continuity of f.

Also $x_n \in \mathrm{Dom}(g)$ and hence $g(x_n) \to g(c)$, (II)

by the continuity of g.

From (I) and (II) and a limit law:

$$(f \cdot g)(x_n) = f(x_n) \cdot g(x_n) \longrightarrow f(c) \cdot g(c)$$

$$= (f \cdot g)(c).$$

$\square$