MATH 131A (Fall 2002, Lecture 1)
Instructor: Roberto Schonmann
Second Midterm Exam

Last Name:

First and Middle Names:

Signature:

UCLA id number (if you are an extension student, say so):

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	Total
Score						

1) (10 points) Define the meaning of "l is a least upper bound of the set $S \subset \mathbb{R}$". (Provide the mathematical precise definition, not just an explanation in words. You can suppose that upper bound has already been defined.)

l is a l.u.b. of S if

a) l is an upper bound of S.

And

b) for any upper bound b of S,

$l \leq b$.

2) (10 points) Suppose that $\{a_n\}$ and $\{b_n\}$ are Cauchy sequences, and for each n define $c_n = a_n \cdot b_n$. Can you conclude that $\{c_n\}$ is also a Cauchy sequence? If the answer is yes, prove it, and if the answer is no, provide a counterexample.

Yes.

$\{a_n\}$ Cauchy $\Longrightarrow$ $a_n \to a$, some $a \in \mathbb{R}$.

$\{b_n\}$ Cauchy $\Longrightarrow$ $b_n \to b$, some $b \in \mathbb{R}$.

Hence,

$c_n = a_n \cdot b_n \longrightarrow a \cdot b$

This means that $\{c_n\}$ converges. But the convergence of $\{c_n\}$ implies that it is Cauchy.

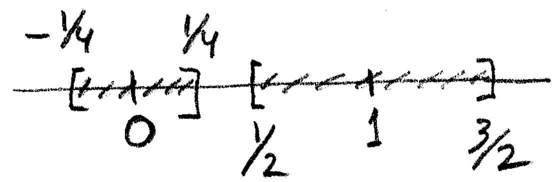
3) (10 points) State the Bolzano-Weierstrass Theorem. (You do not have to prove it.)

Thm (Bolzano - Weierstrass)

Every bounded sequence has a convergent subsequence.

4) (10 points) Provide an example of the following, or explain why this is not possible. A sequence $\{a_n\}$ such that $a_n \to 1$ and with a limit point at 0.

Not possible.



$a_n \to 1 \implies \exists N$ s.t.

$n \geq N \implies a_n \in [1/2, 3/2]$

(take $\varepsilon = \frac{1}{2}$ in the definition of $a_n \to 1$)

Therefore, for $\varepsilon = 1/4$ and N above

$\nexists\, n \geq N$ s.t. $a_n \in [0-\varepsilon, 0+\varepsilon]$.

5) (10 points) Suppose that the function f satisfies $|f(a) - f(b)| \leq |a - b|$, for every $a, b \in \mathrm{Dom}(f)$. Prove that f is a continuous function.

Need to prove that f is continuous at each pt $c \in \mathrm{Dom}(f)$.

Suppose $\{x_n\}$ is s.t. $x_n \in \mathrm{Dom}(f)$ $\forall n$ and $x_n \to c$. Then

$$\forall \varepsilon > 0 \quad \exists N \quad \text{s.t.} \quad n \geq N \Rightarrow |x_n - c| \leq \varepsilon.$$

But then also

$$n \geq N \Rightarrow |f(x_n) - f(c)| \leq |x_n - c| \leq \varepsilon.$$

$\square$