MATH 131A (Fall 2002, Lecture 2)
Instructor: Roberto Schonmann
First Midterm Exam

Last Name:

First and Middle Names:

Signature:

UCLA id number (if you are an extension student, say so):

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	Total
Score						

1) (10 points) Either draw the graph of a one-to-one correspondence between the sets A and B below, or explain why this cannot be done.

$$A = (0,1) \qquad B = (6,8) \cap \mathbb{Q}$$

Cannot be done, because A is not countable, while B is countable (since B is a subset of Q which is countable).

2) (10 points) Prove by induction that $2^1 + 2^2 + ... + 2^n = 2^{n+1} - 2$.

a) Case $n = 1$: $2^1 + 2^2 + \dots + 2^n = 2^1 = 2$

$2^{n+1} - 2 = 2^2 - 2 = 4 - 2 = 2$

b) (Induction argument) Suppose true for n.

Then

$$2^1 + 2^2 + \dots + 2^n + 2^{n+1} = (2^1 + 2^2 + \dots + 2^n) + 2^{n+1}$$

$$= (2^{n+1} - 2) + 2^{n+1} = 2^{n+2} - 2 = 2^{(n+1)+1} - 2$$

So also true for $n+1$.

$\square$

3) (10 points) Define $a_n \to -\infty$.

$\forall M \quad \exists N$ such that

$n \geq N \implies a_n \leq M$.

4) (10 points) Suppose that $a_n \to L$, $b_n \to L$ and $a_n \le c_n \le b_n$ for all n. Show that $c_n \to L$.

Since $a_n \to L$, $\forall \varepsilon > 0$ $\exists N_1$ s.t.

$$N \geq N_1 \Rightarrow a_n \in (L-\varepsilon, L+\varepsilon)$$

Since $b_n \to L$, $\forall \varepsilon > 0$ $\exists N_2$ s.t.

$$N \geq N_2 \Rightarrow b_n \in (L-\varepsilon, L+\varepsilon)$$

Take $N = \max\{N_1, N_2\}$. Then

$$n \geq N \Rightarrow L-\varepsilon < a_n \leq c_n \leq b_n \leq L+\varepsilon$$

$$\Rightarrow c_n \in (L-\varepsilon, L+\varepsilon).$$

$\square$

5) (10 points) Suppose that $a_n \to \infty$ and define $b_n = 1/a_n$. Show that $b_n \to 0$.

Since $a_n \to \infty$,

$\forall\, \varepsilon > 0 \quad \exists\, N$ s.t.

$$n \geq N \implies a_n > 1/\varepsilon \;.$$

But then

$$n \geq N \implies \left\{ \begin{array}{l} a_n > 0 \implies b_n = \dfrac{1}{a_n} > 0 \\ b_n = \dfrac{1}{a_n} < \varepsilon \end{array} \right\}$$

$$\implies 0 < b_n < \varepsilon$$

$$\implies -\varepsilon < b_n < \varepsilon$$

$$\implies |b_n| < \varepsilon .$$

$\square$