MATH 115A (Spring 2006, Lecture 1)
Instructor: Roberto Schonmann
Second Midterm Exam

Last Name:

First and Middle Names: Solutions

Signature:

UCLA id number (if you are an extension student, say so):

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	Total
Score						

1) (10 points) Find the nullity and the rank of the linear transformation $T : I\!R^3 \to I\!R^2$ given by $T(1,0,0) = (1,0)$, $T(0,1,0) = (-1,0)$, and $T(0,0,1) = (2,0)$.

$$T(a,b,c) = (a-b+2c,\ 0)$$

$$N(T) = \{(a,b,c) : T(a,b,c) = 0\} =$$

$$= \{(a,b,c) : a-b+2c = 0\}$$

$$= \{(b-2c, b, c) : b, c \in \mathbb{R}\}$$

nullity $= \dim N(T) = \underline{\underline{2}}$.

Dimension Thm : rank + nullity $= \dim \mathbb{R}^3$

$\therefore$ rank $= 3 - 2 = \underline{\underline{1}}$.

2) (10 points) Let $V = P(\mathbb{R})$, the vector space of polynomials with coefficients in $\mathbb{R}$. Define the linear operators $T, U \in \mathcal{L}(V)$ by $T(f(x)) = f'(x)$ and $U(f(x)) = f''(x)$. Is the set $\{T, U\}$ a linearly independent subset of $\mathcal{L}(V)$? Prove your answer.

Want to check if $aT + bU = 0 \Rightarrow a = b = 0$

$aT + bU = 0$ means $\forall f \in P(\mathbb{R})$, $aTf + bUf = 0$

i.e. $af' + bf'' = 0$

Take $f(x) = x^2$. Then $af'(x) + bf''(x) = 2ax + 2b$

But $2ax + 2b = 0 \quad \forall x \Rightarrow a = b = 0$

So the answer is <u>yes</u>.

3) (10 points) Let V, W and Z be vector spaces over the same field F, and let $T : V \to W$ and $U : W \to Z$ be linear. Prove that $UT : V \to Z$ is linear.

$\forall a \in F,\ x, y \in V$

$$UT(ax+y) \underset{\substack{\uparrow \\ \text{def. } UT}}{=} U(T(ax+y)) \underset{\substack{\uparrow \\ T \text{ linear}}}{=} U(aT(x)+T(y))$$

$$\underset{\substack{\uparrow \\ U \text{ linear}}}{=} aU(T(x)) + U(T(y)) \underset{\substack{\uparrow \\ \text{def } UT}}{=} a\,UT(x) + UT(y).$$

4) (10 points) Let Q be an $n \times n$ invertible matrix. Define $T : M_{n\times n} \to M_{n\times n}$ by $T(A) = QA$. Is T an isomorphism? Prove your answer.

Were only asked to check T invertible :

one-to-one ? $T(A) = T(B) \Rightarrow QA = QB$

$\Rightarrow Q^{-1}QA = Q^{-1}QB$

$\Rightarrow A = B$. yes.

Since T maps $M_{n\times n}$ to itself, one-to-one and onto are equivalent.

So, yes, T is isomorphism.

5) (10 points) Define similar matrices.

Two matrices A, B of same $n \times n$ size are similar if there exists an invertible $n \times n$ matrix Q s.t.

$$B = Q^{-1} A Q .$$