MATH 115A (Spring 2006, Lecture 1)
Instructor: Roberto Schonmann
First Midterm Exam

Last Name:

First and Middle Names: Solutions

Signature:

UCLA id number (if you are an extension student, say so):

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	Total
Score						

1) (10 points) Using only the axioms which define vector spaces, prove that the 0 vector described in (VS3) is unique. State clearly which axioms you use.

Suppose also $x + 0' = x \quad \forall x \in V$. (I)

Want to show $0' = 0$.

From (I) and (V3): $x + 0' = x + 0 \quad \forall x \in V$

Using (V1), $0' + x = 0 + x \quad \forall x \in V$

In case $x = 0$, we obtain $0' + 0 = 0 + 0$

Using (V3) on both sides, $0' = 0$

$\square$

2) (10 points) Is $W = \{(a_1, ..., a_n) \in F^n \mid a_1 + ... + a_n = 0\}$ a subspace of F^n? Prove your answer.

Yes. Enough to check the three properties below:

(a) $0 = (0, \dots, 0)$ satisfies $a_1 + \dots + a_n = 0 + \dots + 0 = 0$, so $0 \in W$

(b) If $x = (a_1, \dots, a_n) \in W$ and $y = (b_1, \dots, b_n) \in W$, then $x + y = (a_1 + b_1, \dots, a_n + b_n)$ satisfies $(a_1 + b_1) + \dots + (a_n + b_n) = (a_1 + \dots + a_n) + (b_1 + \dots + b_n)$

$= 0 + 0 = 0$. So also $x + y \in W$.

(c) If $x = (a_1, \dots, a_n) \in W$ and $c \in F$, then

$cx = (ca_1, \dots, ca_n)$ satisfies

$ca_1 + \dots + ca_n = c(a_1 + \dots + a_n) = c \cdot 0 = 0$.

So also $cx \in W$.

3) (10 points) Define span(S).

$$\text{span}(S) = \{ a_1 v_1 + a_2 v_2 + \cdots + a_n v_n : v_1, \ldots, v_n \in S,$$

$$a_1, \ldots, a_n \in F, \ n = 1, 2, \ldots \}$$

4) (10 points) Suppose that V is a vector space over $I\!R$ and that $\{v, u\} \subset V$ is linearly independent. Does it follow necessarily that also the set $\{v+u, v-u\}$ is linearly independent? Prove your answer.

Suppose $a(v+u) + b(v-u) = 0$

Then $(a+b)v + (a-b)u = 0$

Since $\{v, u\}$ l.i., must have

$a+b = 0$ and $a-b = 0$ (I)

Adding equations in (I): $2a = 0 \therefore a = 0$

From first equation in (I) now: $b = 0 - a = 0$

So $a(v+u) + b(v-u) = 0 \Rightarrow a = b = 0$

This means that $\{v+u, v-u\}$ l.i.

5) (10 points) Find the dimension of the subspace of $\mathcal{F}(\mathbb{R}, \mathbb{R})$ generated by the functions $f(x) = e^x$, $g(x) = x^2 e^x$ and $h(x) = e^x(1 + x^2)$. Prove your answer.

Call $\text{span}\{f, g, h\} = W$

Clearly $h = f + g$. So also $\text{span}\{f, g\} = W$

Let's check that $\{f, g\}$ l.i. :

$$af + bg = 0 \Rightarrow \forall x \in \mathbb{R} \quad ae^x + bx^2 e^x = 0$$

$$\begin{cases} \text{for } x = 0 \Rightarrow a + b \cdot 0 = 0 \Rightarrow a = 0 \\ \text{for } x = 1 \Rightarrow ae + be = 0 \Rightarrow b = -a = 0 \end{cases}$$

So $\{f, g\}$ l.i. . Since it also generates W,

$\{f, g\}$ is basis for W.

So $\dim(W) = 2$

Axioms which define a vector space V over a field F:

(VS 1) $\forall x, y \in V$, $x + y = y + x$.

(VS 2) $\forall x, y, z \in V$, $(x + y) + z = x + (y + z)$.

(VS 3) $\exists 0 \in V$ s. t. $\forall x \in V$, $x + 0 = x$.

(VS 4) $\forall x \in V$, $\exists y \in V$ s. t. $x + y = 0$.

(VS 5) $\forall x \in V$, $1x = x$.

(VS 6) $\forall a, b \in F$, $\forall x \in V$, $(ab)x = a(bx)$.

(VS 7) $\forall a \in F$, $\forall x, y \in V$, $a(x + y) = ax + ay$.

(VS 8) $\forall a, b \in F$, $\forall x \in V$, $(a + b)x = ax + bx$.