MATH 113 (Spring 2004, Lecture 2)
Instructor: Roberto Schonmann
Midterm Exam

Last Name:

First and Middle Names:

Signature:

UCLA id number (if you are an extension student, say so):

Provide the information asked above and write your name on the top of each page using a pen. You should show your work and explain what you are doing; this is more important than just finding the right answer. You can use the blank pages as scratch paper or if you need space to finish the solution to a question. Please, make clear what your solution and answer to each problem is. When you continue on another page indicate this clearly. You are not allowed to sit next to students with whom you have been studying for this exam or to your friends.

Good Luck !

Question	1	2	3	4	5	Total
Score						

1) (10 points) A committee is to be chosen from among 8 seniors, 5 juniors, 2 sophomores and 3 freshmen. If the committee is to have exactly 2 members from a single one of these groups, how many choices are there? (You should provide a numerical answer.)

$$\begin{array}{ccccccc} Se & & J & & So & & F \\ \binom{8}{2} & + & \binom{5}{2} & + & \binom{2}{2} & + & \binom{3}{2} \end{array}$$

$$= \frac{8 \times 7}{2} + \frac{5 \times 4}{2} + 1 + 3$$

$$= 28 + 10 + 1 + 3$$

$$= \boxed{42}$$

2) (10 points) Show that

$$\sum_{k=0}^{\infty} \binom{n}{2k} = 2^{n-1}.$$

$$l.h.s = \binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots$$

We know $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots = 0$

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots = 2^n$$

Adding these get $2 \cdot (l.h.s.) = 2^n$

$$\therefore \quad l.h.s = 2^{n-1} = r.h.s.$$

3) (10 points) Use the notation $T(n,k)$ for the number of ways to place n distinguishable balls in k distinguishable cells, with no cell empty. Explain why

$$T(n+1,k) = kT(n,k) + kT(n,k-1)$$

Balls : $1, 2, 3, \ldots, n, n+1$. k cells

1st operation : Place ball $n+1$ in a cell (k choices)

2nd operation : Place remaining balls $1, 2, \ldots, n$ either

- In cells where $(n+1)$th ball is not ($T(n, k-1)$ choices)

or

- In all the k cells, with at least one such ball in each ($T(n,k)$ choices)

So $T(n+1,k) = k\left(T(n,k-1) + T(n,k)\right)$
$= k\,T(n,k-1) + k\,T(n,k)$

4) (10 points) Find a sequence for which the exponential generating function is

$$H(x) = \frac{2x}{1+x^2}$$

$$H(x) = \frac{2x}{1+x^2} = 2x\,(1 - x^2 + x^4 - x^6 + x^8 - x^{10} + \dots)$$

$$= 2x - 2x^3 + 2x^5 - 2x^7 + 2x^9 - 2x^{11} + \dots$$

$$H(x) = \sum_{k=0}^{\infty} a_k \frac{x^k}{k!}$$

$$\Rightarrow \begin{cases} a_k = 0 & \text{if } k \text{ is even} \\ a_k = k! \cdot 2 \cdot (-1)^{\frac{k-1}{2}} & \text{if } k \text{ is odd} \end{cases}$$

5) (10 points) Three people each roll a die once. In how many ways can the score add up to 17? (You should provide a numerical answer.)

The simplest solution is to list the outcomes:

6, 6, 5 , 5, 5, 6 , 5, 6, 6

$\boxed{\text{Answer : } 3}$