

# CORRECTION TO “FIXED POINTS OF MAPS OF A NON-ASPHERICAL WEDGE”

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Let  $X = P \vee C$  be the wedge of the real projective plane  $P$  and the circle  $C$ . As the example below demonstrates, the statement on page 3 of [1] “Given a map  $f: X \rightarrow X$  we may deform  $f$  by

a homotopy so that  $f_P$ , its restriction to  $P$ , maps  $P$  to itself.” is incorrect. If, instead of an arbitrary self-map of  $X$ , we consider a map of pairs  $f: (X, P) \rightarrow (X, P)$ , the map can be put in the *standard form* defined on that page and then all the results of the paper are correct for such maps of pairs.

To describe the example, represent points  $x$  of the unit 2-sphere  $S^2$  by spherical coordinates  $x = (r = 1, \theta, \phi)$  where  $r$  denotes the radius,  $\theta$  the elevation and  $\phi$  the azimuth. Let  $S^2 = D_+^2 \cup A_+ \cup E \cup A_- \cup D_-^2$  where  $x$  is in  $D_+^2, A_+, E, A_-$  or  $D_-^2$ , if  $\pi/3 < \theta \leq \pi/2, \pi/6 < \theta \leq \pi/3, -\pi/6 \leq \theta \leq \pi/6, -\pi/3 \leq \theta < -\pi/6$  or  $-\pi/2 \leq \theta < -\pi/3$ , respectively. Let  $Y = S_+^2 \cup I_+ \cup S^2 \cup I_- \cup S_-^2$ , where  $S_\pm^2$  are the 2-spheres of radius one in  $\mathbb{R}^3$  with centers, in cartesian coordinates, at  $(\pm 2, 0, \pm 2)$ ,  $I_+$  denotes the points  $(t, 0, 1)$  for  $0 \leq t \leq 2$  and  $I_-$  the points  $(t, 0, -1)$  for  $-2 \leq t \leq 0$ . Define  $\tilde{f}_P: S^2 \rightarrow Y$  in the following manner. For  $x = (1, \theta, \phi) \in A_\pm$ , let

$$\tilde{f}_P(x) = \tilde{f}_P(1, \theta, \phi) = \left(\frac{12\theta}{\pi} - 2, 0, \pm 1\right) \in \mathbb{R}^3$$

in cartesian coordinates. For  $(1, \theta, \phi) \in E$ , set  $\tilde{f}_P(1, \theta, \phi) = (1, 3\theta, \phi)$ . Let  $\rho_\pm = (1, \pm\pi/2, 0) \in S^2$  be the poles and define  $K_\pm: D_\pm^2 \rightarrow S^2 - \rho_\mp$  by

$$K_\pm(x) = K_\pm(1, \theta, \phi) = (1, 6\theta \mp 5\pi/2, \phi).$$

Returning to cartesian coordinates, define  $T_\pm: S^2 \rightarrow S_\pm^2$  by

$$T_\pm(x_1, x_2, x_3) = (x_1 \pm 2, x_2, x_3 \pm 2).$$

We complete the definition of  $\tilde{f}_P: S^2 \rightarrow Y$  by setting  $\tilde{f}_P(x) = T_\pm K_\pm$  for  $x \in D_\pm^2$ . Note that  $(\tilde{f}_P)_*: H_2(S^2, \mathbb{Z}/2\mathbb{Z}) \rightarrow H_2(Y, \mathbb{Z}/2\mathbb{Z})$  such that  $(\tilde{f}_P)_*(1) = (1, 1, 1)$ . We may embed  $Y$  in the universal covering space  $p: \tilde{X} \rightarrow X$  because  $\tilde{X}$  is an infinite tree with a 2-sphere replacing each vertex in such a way that two edges are attached at each of two antipodal points. The embedding induces a monomorphism of homology. The map  $\tilde{f}_P$  has been defined so that if  $x, -x$  are antipodal points of  $S^2$ , then  $p\tilde{f}_P(x) = p\tilde{f}_P(-x)$  and therefore  $\tilde{f}_P$  induces a map  $f_P: P \rightarrow X$ . If  $f_P$  were homotopic to a map  $g_P: P \rightarrow P \subseteq X$ , then the homotopy would lift to cover  $g_P$  by a map  $\tilde{g}_P: S^2 \rightarrow \tilde{X}$  which sends  $S^2$  to a single 2-sphere in  $\tilde{X}$ . Therefore the image of  $(\tilde{g}_P)_*: H_2(S^2, \mathbb{Z}/2\mathbb{Z}) \rightarrow H_2(\tilde{X}, \mathbb{Z}/2\mathbb{Z})$  would be either trivial or a single generator of  $H_2(\tilde{X}, \mathbb{Z}/2\mathbb{Z})$ . On the other hand, the image of  $(\tilde{f}_P)_*$  in  $H_2(\tilde{X}, \mathbb{Z}/2\mathbb{Z})$  is nontrivial for three generators, so no such homotopy can exist. Therefore, if  $f: X \rightarrow X = P \vee C$  is a map

whose restriction to  $P$  is the map  $f_P$  defined above, then it cannot be homotoped to a map that takes  $P$  to itself.

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## References

- [1] S. Kim, R. Brown, A. Ericksen, N. Khamsemanan and K. Merrill, “Fixed points of maps of a nonaspherical wedge”, *Fixed Point Theory and Applications*, vol. 2009, Article ID 531037, 18 pages, 2009.