

1. (15 points) Prove that

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

2. (20 points) Bismuth-210 has a half-life of 5 days. A sample has a mass of 800 mg. When is the mass reduced to 100 mg?

3. (20 points) Find the limits.

$$(a) \quad \lim_{x \rightarrow \pi} \frac{\tan x}{x - \pi}$$

$$(b) \quad \lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$$

4. (20 points) The Error Bound Theorem states that for  $T_n(x)$  the  $n$ -th order Taylor polynomial for  $f(x)$  at  $x = a$  and  $x$  in the interval  $[x_1, x_2]$  containing  $a$ ,

$$|f(x) - T_n(x)| \leq K \frac{|x - a|^{n+1}}{(n + 1)!}$$

if  $|f^{(n+1)}(x)| \leq K$  for all  $x$  in  $[x_1, x_2]$ . Show that the error bound for the 2nd-order Taylor polynomial for  $f(x) = \ln(1 + x)$  at  $x = 1$  on the interval  $[\frac{1}{2}, \frac{3}{2}]$  is  $\frac{1}{81}$ .

5. (25 points) Evaluate the integrals.

$$(a) \quad \int \sin^2 2x \, dx$$

$$(b) \quad \int \sin^{-1} 2x \, dx$$