1. [10 points] Simplify the following expression so that your answer is a whole number

$$\frac{\ln(\frac{1}{8}) - \ln(16)}{\ln(\frac{1}{2})}$$

2. [20 points] Differentiate

$$(a) \qquad f(x) = \arctan(\log_3(x^2+1))$$

$$(b) \qquad f(x) = (\cosh x)^x$$

3. [20 points] Calculate the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sinh x$ from 0 to 1.

4. [15 points] (a) Find the domain and range of the function $\arcsin(\sin x)$.

(b) Find $\arcsin(\sin(10\pi/3))$.

5. [15 points] Find the average value of the function $\sec^3 x$ on $[0, \pi/4]$.

6. [25 points] Evaluate the integrals.

$$(a) \qquad \int_0^\infty x3^{-x^2}\,dx$$

$$(b) \qquad \int_0^1 \ln 2x\,dx$$

7. [20 points] Find the arc length of the curve $y = \ln(\cos x)$ for x in $[0, \pi/4]$.

8. [20 points]

(a) Let $f(x) = \sin(\sin x)$. Show that $|f''(x)| \leq 2$ for all x in $[0, 1]$.

(b) Using part (a), determine how large n must be so that the midpoint approximation M_n to

$$\int_0^1 \sin(\sin x)\, dx$$

is accurate to within 10^{-4}.

9. [25 points] Newton's Law of Cooling says that the rate of change of the temperature of a cooling body is proportional to the difference between the temperature $T(t)$ at time t of the cooling body and the constant temperature T_m of the surrounding air, that is,

$$\frac{dT}{dt} = k(T(t) - T_m)$$

(a) Separate the variables and find the general solution to this differential equation, showing your work.

A cup of coffee has the initial temperature of 90^oC. It is brought into a 20^oC. room and, ten minutes later, the temperature of the coffee is 50^oC.

(b) Find the function $T(t)$ that given the temperature of the coffee at time t.

10. [30 points] Integrate

$$(a) \qquad \int \frac{dx}{x^2(x^2+1)}$$

$$(b) \qquad \int \frac{\sqrt{1-x^2}}{x^2}\, dx$$