

1. Given the function

$$f(x) = x^4 - 2x^3 - 12x^2 + 5$$

determine where the function is concave upward and concave downward, and find the points of inflection, if there are any.

2. Consider the function

$$f(x) = \frac{\sqrt{2x^2 - 1}}{x}.$$

(a) Determine the domain of  $f(x)$ .

(b) Find the horizontal asymptotes of  $f(x)$ , if there are any.

3. As the Sun sets, the angle of elevation of the Sun above the horizon is decreasing at the rate of  $\frac{1}{4}$  radian/hr. How fast is the shadow cast by a 400-foot-tall building increasing when the angle of elevation of the Sun is  $\pi/4$  radians. Note: the angle of elevation of the Sun is the angle from the ground - assumed flat - up to the Sun.

4. Find all the local maxima and local minima of the function

$$f(x) = x^{2/3}(1-x)^2.$$

5. Suppose that a function  $f(x)$  has second derivative  $f''(x)$  for all real numbers  $x$ .

(a) Prove that there exists a point  $c$  in  $(0, 1)$  such that

$$f(1) - f(0) = f'(c).$$

(b) Prove that if  $|f''(x)| \leq 1$  for all  $x$  in  $(0, 1)$ , then

$$|f(1) - f(0) - f'(0)| < 1.$$