

1. Differentiate

$$(a) \quad f(x) = \frac{2x+1}{1+\sec x}$$

$$(b) \quad f(x) = \sqrt{x - 2\sqrt{x^2 - 1}}$$

$$(a) \quad f'(x) = \frac{2(1+\sec x) - (2x+1)(\sec x \tan x)}{(1+\sec x)^2}$$

$$(b) \quad f(x) = (x - 2(x^2 - 1)^{\frac{1}{2}})^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(x - 2(x^2 - 1)^{\frac{1}{2}})^{-\frac{1}{2}} \left(1 - 2\left(\frac{1}{2}\right)(x^2 - 1)^{-\frac{1}{2}}(2x) \right)$$

2. Find all the values of x for which the tangent lines to the graph of

$$f(x) = x^2 - x + 1$$

at $(x, f(x))$ pass through the point $(1, -3)$.

$$f'(x) = 2x - 1$$

$$2a - 1 = \frac{(a^2 - a + 1) - (-3)}{a - 1}$$

$$2a^2 - a - 2a + 1 = a^2 - a + 1 + 3$$

$$a^2 - 2a - 3 = 0$$

$$(a - 3)(a + 1) = 0$$

$$x = -1, 3$$

3. Evaluate the limits, if they exist.

$$(a) \quad \lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 2x}{x^2 - x - 2}$$

$$(b) \quad \lim_{x \rightarrow 0} x \cot x$$

$$\begin{aligned} (a) \quad \lim_{x \rightarrow 2} \frac{x(x^2 - 3x + 2)}{x^2 - x - 2} &= \lim_{x \rightarrow 2} \frac{x(\cancel{x-2})(x-1)}{(\cancel{x-2})(x+1)} \\ &= \frac{2(2-1)}{2+1} = 2/3 \end{aligned}$$

$$\begin{aligned} (b) \quad \lim_{x \rightarrow 0} \frac{x \cos x}{\sin x} &= \lim_{x \rightarrow 0} \frac{\cos x}{\frac{\sin x}{x}} \\ &= \frac{\lim_{x \rightarrow 0} \cos x}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} = \frac{1}{1} = 1 \end{aligned}$$

4. Find the equation of the tangent line to the curve

$$(x^2 + y^2)^2 = 2(x^2y + 1)$$

at $(-1, 1)$.

$$\cancel{2}(x^2 + y^2)(2x + 2y \frac{dy}{dx}) = \cancel{2}(2xy + x^2 \frac{dy}{dx})$$

$$(1+1)(-2+2 \frac{dy}{dx}) = -2 + \frac{dy}{dx}$$

$$-4 + 4 \frac{dy}{dx} = -2 \frac{dy}{dx} \quad \frac{dy}{dx} = \frac{2}{3}$$

$$y - 1 = \frac{2}{3}(x + 1)$$

5. Use the definition of the derivative to find the derivative of

$$f(x) = \sqrt{2x^2 + 1}$$

$$f'(a) = \lim_{x \rightarrow a} \frac{\sqrt{2x^2 + 1} - \sqrt{2a^2 + 1}}{x - a} \cdot \frac{\sqrt{2x^2 + 1} + \sqrt{2a^2 + 1}}{\sqrt{2x^2 + 1} + \sqrt{2a^2 + 1}}$$

$$= \lim_{x \rightarrow a} \frac{(2x^2 + 1) - (2a^2 + 1)}{(x - a)(\sqrt{2x^2 + 1} + \sqrt{2a^2 + 1})}$$

$$= \lim_{x \rightarrow a} \frac{2(x^2 - a^2)}{(x - a)(\sqrt{2x^2 + 1} + \sqrt{2a^2 + 1})}$$

$$= \lim_{x \rightarrow a} \frac{2(x - a)(x + a)}{(x - a)(\sqrt{2x^2 + 1} + \sqrt{2a^2 + 1})}$$

$$= \frac{2(2a)}{2\sqrt{2a^2 + 1}}$$