1. Let

$$f(x) = \tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 - \tfrac{3}{2}x^2 + 1.$$

(a) Find the critical numbers of f.

(b) Find the intervals of increase and decrease of f.

2. Calculate the limit, if it exists.

$$\lim_{x \to 0} \frac{\tan 4x}{x}.$$

3. Calculate

(a) the slope of the tangent to the curve

$$x \sin y = x + 1$$

at $(-1, 0)$.

(b) the derivative of

$$f(x) = \int_{-1}^{x^2} \sqrt{1 + t^4}\, dt$$

at $x = 2$.

4. A line of negative slope through $(3, 5)$ bounds a triangular region of the first quadrant of the plane.

(a) Write the area of the triangle as a function of the slope of the line.

(b) Find the slope that makes the area of the triangular region as small as possible.

5. Evaluate

$$(a) \qquad \int_0^{\pi} \sqrt{x} + \sin x\, dx$$

$$(b) \qquad \int_{-2}^{0} |x + 1|\, dx$$

6. Find the value of x such that the tangent line to the curve $y = x^3 + 1$ at x passes through the point $(0, -1)$.

7. Evaluate the indefinite integrals

$$(a) \qquad \int \frac{x}{(x^2+1)^2}\,dx$$

$$(b) \qquad \int \sec^3 x \tan x\,dx$$

8. Prove that if $f(x)$ is continuous on $[a, b]$ and $f(a) < f(b)$, then there exists c in $[a, b]$ such that

$$f(c) = \frac{f(a) + f(b)}{2}.$$

9. Find the value of a for which

$$\lim_{x \to 2} \frac{x^2 - 3x + a}{x - 2}$$

exists and evaluate the limit.

10. Let A be the region in the first quadrant bounded by the curves $x = 1, y = 6 - x$ and $y = x^2$. Write the volumes of the following solids as definite integrals but do NOT evaluate the integrals.

(a) The solid formed by rotating A about the line $y = 0$.

(b) The solid formed by rotating A about the line $x = 1$.