

1. Find the (absolute) maximum and minimum of

$$f(x) = \frac{\sin x + \cos x}{2}$$

on the interval $[0, \pi]$, showing your work.

2. Use the Mean Value Theorem to prove that if $f(1) = 3$ and $f'(x) < -1$ for all $x \geq 0$, then $f(4)$ is negative.

3. A function is defined by

$$f(x) = \frac{x}{\sqrt{2x^2 + 1}}.$$

(a) Find the horizontal asymptotes of $f(x)$, if it has any.

(b) Given that

$$f'(x) = \frac{1}{\sqrt{(2x^2 + 1)^3}},$$

determine the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down.

4. Given that

$$f''(x) = \sin x + 6x,$$

$f'(\pi) = 1$ and $f(\pi) = 0$, find $f(x)$.

5. A kite 100 feet above the ground moves horizontally at a rate of 8 feet per second. At what rate is the angle θ between the kite string and the (horizontal) ground changing when the length of the string is 200 feet? (Note: Think of the kite string as a straight line from a point on the ground to the kite.)