

5. Use the definition of the derivative to show that the derivative of  $f(x) = \cos x$  is  $f'(x) = -\sin x$ . You will need the trigonometric identity of the form

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

and limit theorems from the text as well as limit laws.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos x (\cos h - 1) - \sin x \sin h}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos x (\cos h - 1)}{h} - \lim_{h \rightarrow 0} \frac{\sin x \sin h}{h} \\ &= \lim_{h \rightarrow 0} \cos x \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} - \lim_{h \rightarrow 0} \sin x \lim_{h \rightarrow 0} \frac{\sin h}{h} \\ &= (\cos x)(0) - (\sin x)(1) = -\sin x \end{aligned}$$