1. Differentiate

$$(a) \qquad f(x) = \frac{2x+1}{1+\sec x}$$

$$(b) \qquad f(x) = \sqrt{x - 2\sqrt{x^2-1}}$$

(a) $f'(x) = \dfrac{2(1+\sec x) - (2x+1)(\sec x \tan x)}{(1+\sec x)^2}$

(b) $f(x) = (x - 2(x^2-1)^{1/2})^{1/2}$

$f'(x) = \frac{1}{2}(x-2(x^2-1)^{1/2})^{-1/2}(1-2(\frac{1}{2})(x^2-1)^{-1/2}(2x))$

2. Find all the values of x for which the tangent lines to the graph of

$$f(x) = x^2 - x + 1$$

at $(x, f(x))$ pass through the origin.

$y - 0 = m(x - 0)$

$m = f'(x) = 2x - 1$

$y = x^2 - x + 1$

$x^2 - x + 1 = (2x - 1)(x) = 2x^2 - x$

$-x^2 + 1 = 0 \qquad x = +1, -1$

3. Evaluate the limits, if they exist.

$$(a) \qquad \lim_{x\to 2} \frac{x^3 - 3x^2 + 2x}{x^2 - x - 2}$$

$$(b) \qquad \lim_{x\to 0} x\cot x$$

(a) $$\lim_{x\to 2} \frac{x(x^2-3x+2)}{x^2-x-2} = \lim_{x\to 2} \frac{x\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x+1)}$$

$$= \frac{2(2-1)}{2+1} = 2/3$$

(b) $$\lim_{x\to 0} \frac{x\cos x}{\sin x} = \lim_{x\to 0} \frac{\cos x}{\frac{\sin x}{x}}$$

$$= \frac{\lim\limits_{x\to 0} \cos x}{\lim\limits_{x\to 0} \frac{\sin x}{x}} = \frac{1}{1} = 1$$

4. Find the equation of the tangent line to the curve

$$(x^2+y^2)^2 = 2(x^2y+1)$$

at $(-1,1)$.

$$\cancel{2}(x^2+y^2)(2x+2y\frac{dy}{dx}) = \cancel{2}(2xy + x^2\frac{dy}{dx})$$

$$(1+1)(-2+2\frac{dy}{dx}) = -2 + \frac{dy}{dx}$$

$$-4+4\frac{dy}{dx} = -2\frac{dy}{dx} \qquad \frac{dy}{dx} = 2/3$$

$$y-1 = \frac{2}{3}(x+1)$$

5. Use the **definition** of the derivative to find the derivative of

$$f(x) = \sqrt{2x^2 + 1}$$

$$f'(a) = \lim_{x \to a} \frac{\sqrt{2x^2+1} - \sqrt{2a^2+1}}{x - a} \cdot \frac{\sqrt{2x^2+1} + \sqrt{2a^2+1}}{\sqrt{2x^2+1} + \sqrt{2a^2+1}}$$

$$= \lim_{x \to a} \frac{(2x^2+1) - (2a^2+1)}{(x-a)(\sqrt{2x^2+1} + \sqrt{2a^2+1}}$$

$$= \lim_{x \to a} \frac{2(x^2-a^2)}{(x-a)(\sqrt{2x^2+1} + \sqrt{2a^2+1})}$$

$$= \lim_{x \to a} \frac{2\cancel{(x-a)}(x+a)}{\cancel{(x-a)}(\sqrt{2x^2+1} + \sqrt{2a^2+1})}$$

$$= \frac{2(2a)}{2\sqrt{2a^2+1}}$$