

1. (a) Differentiate

$$f(x) = \sqrt{\sin^3 x + 1}$$

- (b) Calculate the **second** derivative of

$$f(x) = \frac{x}{x^2 + 1}$$

2. Find all the values of  $x$  for which the tangent line to the curve

$$xy^2 = y + 3x^3$$

is horizontal.

3. Prove that

$$\frac{\cos x}{\sin x + 2} = x$$

for some  $x$  in  $[0, \pi/2]$ .

4. Find the value of  $a$  for which the limit

$$\lim_{x \rightarrow 2} \frac{x^2 - \frac{5}{3}x + a}{x - 2}$$

exists and calculate the limit.

5. (a) Write the definition of the derivative  $g'(x)$  of a function  $g(x)$  that is the limit of a quotient in which the denominator is  $h$ .

(b) Use the definition of part (a) to prove that if  $g(x) = xf(x)$ , then

$$g'(x) = xf'(x) + f(x).$$

(Do not use Leibniz' Rule.)