Midterm (practice version - NOT GRADED), Math 170S, Spring 2019
Instructor: Elizaveta Rebrova

Printed name: ________________________________
Signed name: ________________________________
Student ID number: ____________________________

Instructions:

• Read problems very carefully. If you have any questions please ask.

• You should show all the details of your solution, the correct final answer alone gets a very small part of the credit. If you do not have time to write a proper solution, always attach your drafts and try to make them easier for us to follow.

• You final answers do not need to be completely simplified (unless otherwise stated), e.g. a sum of several decimal fractions would be completely fine. Ask if in doubt.

• Calculators and books are not allowed. Phones, computers and any other communication devices are not allowed too.

• You can bring a letter size piece of paper with formulas and anything helpful for you (please be reasonable and do not squeeze out the whole textbook inside one page ;)). Please bring your own writing utensils. We provide all necessary distribution tables and extra draft paper. If you need more draft paper please raise your hand.

• Please have your ID ready so we can check it without disturbing your work.

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1. For this problem: provide short explanations only.

In a certain political campaign, more than 50% supportive votes will secure a win of a candidate. One candidate has a poll taken at random among the voting population, and computed confidence intervals (for the fraction of voters in favor) with confidence level $1 - \alpha$: a one-sided interval $[a, b]$ for the lower estimate; and a two-sided interval $[c, d]$.

(a) (2 points) What is $b$?

(b) (2 points) What can be bigger, $a$ or $c$?

(c) (2 points) What condition must be satisfied so that a candidate feels confident about her win? (with probability $1 - \alpha$)

(d) (2 points) (Not about these elections any more :) ) $X_1, \ldots, X_n$ is a sample from the distribution $Unif[0, \theta]$. Is the estimator $\hat{\theta} = 2\bar{X}$ biased or not?

(e) (2 points) True or false: in the example above, if $X_{(n)}$ is a sufficient statistic, then $X_{(n)}^2$ is also sufficient?

Hint: what are the possible values of $\theta$?
2. (10 points) You have a biased coin with \( \Theta \) being a probability of heads. You believe that \( \Theta \) has pdf

\[
f_{\Theta}(\theta) = 2 - 4|0.5 - \theta| \quad \text{for} \ 0 \leq \theta \leq 1
\]

Find the MAP estimate of \( \Theta \), if you observed exactly one heads in \( n \) experiments? (Get a general formula for any \( n = 1, 2, \ldots \) )

Check your answer: what is the best \( \hat{\theta} \) you got

- for 5 experiments (coin flips)?
- for 2 experiments (coin flips)?
3. Consider a sequence of independent coin tosses, and let $\theta$ be the probability of heads at each toss.

(a) (5 points) Let $k$ be a fixed number, and $N$ is the number of tosses until $k$-th head occurs. Find maximum likelihood estimator of $\theta$ (depending on $N$).

(b) (5 points) Now let $n$ be a fixed number, and $K$ is the number of heads in $n$ tosses. Find maximum likelihood estimator of $\theta$ (depending on $K$).
4. (10 points) Let $X_1, X_2, \ldots, X_n$ are samples taken from the uniform distribution

$$Unif([\theta, \theta + 1]).$$

What is the length of a confidence interval with confidence coefficient $1 - \alpha$ for the estimate $X_{(1)}$ for parameter $\theta$?
5. (10 points) Let $X_1, X_2, \ldots, X_n$ be samples taken from some distribution with unknown density function. However, you know that $\mathbb{E}X = \frac{\alpha}{\beta}$ and $\text{Var } X = \frac{\alpha}{\beta^2}$, where $\alpha$ and $\beta$ are two parameters of the density function (and there is no third parameter). What method could we use to find good estimates for $\alpha$ and $\beta$? What are these estimates?