**Review class 1 - March 14**

**Problem 1.** Assume, the observations $X_1, X_2, \ldots, X_n$ are iid. normal distributed random variables with unknown mean $\theta$ and known variance 4. You observe $n = 16$ many variables with the empirical mean 1.45 and a sample variance of 0.51$^2$.

a) Determine a 90% two-sided confidence interval for the mean.

b) How can we decide on the hypothesis $H_0 : \mu = 2$ vs $H_1 : \mu \neq 2$ on the significance level 10%, using just the answer for part a) and no additional computations?

c) Now assume that, instead of using the sample variance, you know that the variance of the observed random variables $X_1, \ldots, X_n$ is 0.5$^2$? What is the confidence of the confidence interval $[5/4, 13/8]$?

**Problem 2.** A random variable $X$ is characterized by a normal density with mean $\mu_0 = 20$ and variance is either $\sigma_0^2 = 16$ (hypothesis $H_0$) or $\sigma_1^2 = 25$ (hypothesis $H_1$). We want to test $H_0$ against $H_1$, using four sample values $x_1, x_2, x_3, x_4$ and a rejection region of the form

$R = \{x|x_1 + x_2 + x_3 + x_4 > \gamma\}$

for some scalar $\gamma$. Determine the value of $\gamma$ so that the probability of false rejection is 0.05. What is the corresponding probability of the false acceptance?

**Problem 3.** based on 8.6-7 Hogg. Let $X_1, \ldots, X_{10}$ be a random sample from Poisson distribution with mean $\mu$.

a) Show that a uniformly most powerful critical region for testing $H_0 : \mu = 0.5$ against $H_1 : \mu > 0.5$ can be defined using sample mean statistic.

b) Find a general formula for all uniformly best critical regions.

c) Find a uniformly most powerful critical region of the size $\alpha = 0.068$. Hint: if $X_i \sim Poi(\mu)$ and independent, then $\sum_{i=1}^{m} X_i \sim Poi(m\mu)$

d) Sketch the power function of this test.

Hint: see a similar example on the class website http://www.math.ucla.edu/~rebrova/onneyman-pearson.pdf and also example 8.6-3 on page 403

**Problem 4.** 8.2-17 (Hogg) Consider the distributions $N(\mu_X, 400)$ and $N(\mu_Y, 225)$. Let $\theta = \mu_X - \mu_Y$. Say $\bar{x}$ and $\bar{y}$ denote the observed means of two independent random samples, each of size $n$, from the respective distributions. Say we reject $H_0 : \theta = 0$ and accept $H_1 : \theta > 0$ if $\bar{x} - \bar{y} \geq c$. Let $K(\theta)$ be the power function of the test. Find $n$ and $c$ so that $K(0) = 0.05$ and $K(10) = 0.9$ approximately.

**Problem 5.** Can $p$-value of some test be equal to 0.5? 2? -1?

Suppose $p$-value is equal to 0.3 and the probability to reject $H_0$ under some decision rule is 0.5. Do we accept or reject $H_0$ if it is additionally known that the probability that $H_0$ is false is 0.1? Explain why only the first sentence is not enough to give a conclusive answer.