Math 170S - Homework 4 - due May 2

- Numbers refer to the exercise numbers in the Hogg, Tanis, Zimmerman textbook.
- Show all your work.
- For the definitions of MAP and conditional expectation estimates, and other formulas related to the Bayesian estimation, check out [http://www.math.ucla.edu/~rebrova/bayesian.pdf](http://www.math.ucla.edu/~rebrova/bayesian.pdf)

Problem 1. Assume that the observation $X$ is normal distributed with unknown mean $\theta$ and known variance 4. Assume that the prior $f_\theta$ is normal with mean 2 and variance 2. You observe $X = 2$. Calculate the posterior distribution of $\theta$. What is the MAP estimate of $\theta$? What is the conditional expectation estimate of $\theta$?

Problem 2. We have 2 boxes, each containing 3 balls. Box number 1 contains one black and two white balls; box number 2 contains two black and one white ball.

Our friend chooses one of the boxes at random, probability of choosing box number 1 is $p$. Then he takes one ball from a chosen box (each of three balls can be taken chosen equally likely), and it turns out to be white.

We are going to find MAP estimate for the parameter $\theta = 1$ or 2 to conclude which box was chosen ($\theta$ is the number of the box). For what values of $p$ we can conclude that box number 1 was chosen? For what values of $p$ we can conclude that box number 2 was chosen?

Problem 3. You observe 21 data points measuring the temperature at UCLA on a daily basis. The results yielded a sample mean of 72.3 and a sample variance of 5.6 degrees Fahrenheit.

a) Use Student’s distribution to calculate a 90% confidence interval for the mean.
b) Calculate the approximate confidence of the interval [70.5, 73] for the mean temperature.
c) Calculate a one sided 99 % confidence interval that gives an upper bound on the mean temperature.
d) Given you know that the real variance is 5 degrees, what would be the answers to part (a), (b) and (c) using the normal distribution?

Problem 4. 7.1-16

Problem 5. 7.1-17

Problem 6. 7.2-2

Problem 7. 7.2-4