Math 170S - Homework 3 - due April 25

- Numbers refer to the exercise numbers in the Hogg, Tanis, Zimmerman textbook.
- Unless specified otherwise, do all the subparts of a problem.
- If there are additional questions after the number of the problem, you should answer them too.
- Show all your work.

Problem 1. Let \( X_1, \ldots, X_n \) be the sample taken from uniform distribution on the interval \([-\theta, \theta]\). Find maximum likelihood estimator for the parameter \( \theta \).

Problem 2. 6.5-2

Problem 3. 6.5-4

Problem 4. 6.7-2

Problem 5. 6.7-4

Problem 6. Gamma distribution is defined by its density function

\[
f(t; \theta, a) = \frac{\theta^a}{\Gamma(a)} t^{a-1} e^{-\theta t}
\]

when \( t \geq 0 \) and 0 otherwise. Here, \( \theta \) is unknown parameter, \( a \) is a known parameter \( \Gamma(a) \) is a normalization constant.

a) Check that Gamma distribution belongs to the exponential family of distributions.

b) Use this fact to give a sufficient statistic for \( \theta \).

c) Give examples of two more different sufficient statistics for \( \theta \).

Problem 7. a) Show that \( \sum_{i=1}^n X_i \) and \( \sum_{i=1}^n X_i^2 \) are jointly sufficient statistics for two unknown parameters of the normal distribution \( N(\theta_1, \theta_2) \) (based on the data sample \( X_1, \ldots, X_n \)) in two ways: by factorization theorem and using the property of the exponential family.

b) Show that \( \bar{X} \) and \( s^2 \) are jointly sufficient statistics for the same distribution.

c) Give yet another example of a couple of jointly sufficient statistics.

Hint: Example 6.7-5 in Hogg et al. Anyway, make sure to include detailed solution into your homework (not just refer to the aforementioned example). If you claim that something is a sufficient statistic as a function of another sufficient statistic, have you checked that this function is invertible?