Math 170S - Homework 1 - due January 15

Review main concepts, definitions, and examples from 170E, including

- random variable, discrete and continuous distributions, mean, variance, covariance, correlation coefficient
- PMF (probability mass function), PDF (probability density function) and CDF (cumulative distribution function). Difference and connections between them.
- Independence and related properties
- Conditional probability and expectation
- Total probability and Bayes theorem
- Joint and marginal PDFs
- Standard examples of distributions, their interpretations, PDFs (or PMFs), means and variances, including Uniform (discrete and continuous), Bernoulli, Binomial, Poisson, Geometric, Gaussian (= Normal), Exponential
- Moment generating functions
- Central limit theorem

Disclaimer: This is a review homework, based on the material from probability class (only the last problem slightly touches the material of 6.1). You are encouraged to use the textbook Chapters 1-5, collaborate on all the problems (although the write-up of the solutions must be your own) and talk to me if you feel that some of the material is not familiar to you.

Solve the following problems. Unless specified otherwise, show all your work.

Problem 1. (a) Let $X$ be a Binomial random variable such that $E(X) = 4$ and $Var(X) = 2$. Find the parameters of $X$.
(b) Let $X$ be a standard normal random variable. Write down one function $f(t)$ so that the random variable $Y = f(X)$ is normal with mean $a$ and variance $b$.

Problem 2. Let $F_X(t)$ be the cumulative distribution function (CDF) of a continuous random variable $X$ and let $Y = -X$. Express the CDF of $Y$ terms of $F_X(t)$.

Problem 3. Let $Y$ be uniform on \{0, 1, \ldots, 10\} and $Z$ be uniform on $[0, 10]$.
- Let $X_1 = \max(5, \min(Y, 7))$. Find the CDF of $X_1$.
- Compute $Var(X_1)$.
- Let $X_2 = \max(5, \min(Z, 7))$. Find the CDF of $X_2$.
- What kind of random variable (discrete, continuous, or neither) is $X_1$? What about $X_2$? Briefly explain your answer.
Problem 4. A store sells three types of shoes: running shoes (40% of all shoes in the store), hiking shoes (30%) and raining shoes (30%). It is known that 10% of running shoes are water proof; 70% of hiking shoes are water proof; all raining shoes are, of course, water proof. If a selected shoe turns out to be water proof, what is the probability that it is a hiking shoe?

Problem 5. A food store just got a delivery of 10000 potatoes. It is known that each potato is rotten with probability 0.1. What is the expected number of healthy potatoes? Use the normal approximation to estimate the probability that there are at least 8970 healthy potatoes.

Problem 6. Let the random variables $X$ and $Y$ be jointly continuous with the joint PDF

$$f_{X,Y}(x, y) = \begin{cases} ke^{-(ax+by)}, & \text{if } x > 0, y > 0, \\ 0, & \text{otherwise}, \end{cases}$$

where $a, b > 0$ and $k$ is a constant.

(a) Find $k$.

(b) Are $X$ and $Y$ independent?

Problem 7. If $X \sim Binom(10, p)$ and $p \sim Unif[0, 1]$, find $\mathbb{P}(X = 1)$.

Problem 8. Using table Va-b from Appendix B of the textbook, explain percentages that appear in the Empirical Rule (68%, 95% and 99.7%, see page 229), in the case when your data is well approximated by the Normal distribution $N(0, 1)$.