

### Assignment 1 – due Monday, August 17

1. Suppose that  $(\chi, d)$  is a metric space and  $\{x_n\}_{n=1}^{\infty}$  is a sequence in  $\chi$  converging to  $x$ . If  $y \in \chi$ , prove that  $d(x_n, y)$  converges to  $d(x, y)$ .
2. Let  $K$  be a closed subset and  $F$  be a closed subset in the metric space  $\chi$ . If  $K \cap F = \emptyset$ , is it necessarily true that

$$0 < \inf\{d(x, y) : x \in K, y \in F\}?$$

.

3. Are there infinite compact subsets of  $\mathbb{Q}$ ?
4. Prove that the space of continuous functions on the closed interval  $[0, 1]$  with the metric

$$d(f, g) = \sup_{x \in [0, 1]} |f(x) - g(x)|$$

is a complete metric space.

5. Consider the set of real-valued functions on  $[0, 1]$  such that

$$|f(x) - f(y)| \leq |x - y| \text{ and } \int_0^1 f(x) dx = 1.$$

Is this a compact subset of the metric space in problem 4?

6. Let  $\{\mathcal{O}_j\}_{j=1}^n$  be a finite open cover of a compact metric space  $\chi$ .
  - a) Show that there is an  $\epsilon > 0$  such that for every  $x \in \chi$  the ball  $B(x, \epsilon)$  is contained in at least one of the  $\mathcal{O}_j$ 's.
  - b) Show that if none of the  $\mathcal{O}_j$ 's equals  $\chi$ , then there is a largest  $\epsilon$  with the property in part a).