Math 33B-2: Review Problems, Fall 2004

The remarks like "(Section 2.2 #21)" refer to where some of these problems occur in Boyce and DiPrima Edition 7 – so you can check answers. "almost" means that the problem is nearly the same as the one referenced.

1. (Section 2.2 #21) Find the solution in explicit form for

$$\frac{dy}{dx} = \frac{3x^2+1}{3y^2-6y} \tag{0.1}$$
$$y(0) = 1, \tag{0.2}$$

   and determine the interval in which the solution is valid.

2. Find the solution for the initial value problem:

$$y' - \frac{3}{2}y = te^t, \tag{0.3}$$
$$y(0) = y_0. \tag{0.4}$$

   where y_0 is an arbitrary parameter.

3. (almost Section 2.1 #15)Find the solution for the initial value problem:

$$ty' + 2y = t^2 - t + 1, t > 0 \tag{0.5}$$
$$y(\pi) = 1. \tag{0.6}$$

4. Consider the initial value problem:

$$y'' + (3-\alpha)y' + \frac{1}{4}\alpha(\alpha-3)y = 0, \tag{0.7}$$
$$y(0) = 1, \qquad y'(0) = 2, \tag{0.8}$$

   where α is a parameter.

(a) Find the characteristic equation for (0.7).

(b) Determine the conditions on α when the characteristic equation of (0.7) has two

real unequal roots, two real equal roots or two complex conjugate roots.

(c) Take α= 4. Find the solution for equation (0.7) satisfying (0.8).

5. Consider the initial value problem:

$$t^2y'' - 2y = 0, \tag{0.9}$$
$$y(1) = 1, \qquad y'(1) = 2. \tag{0.10}$$

Given that $y_1(t)$=t^2 and $y_2(t)$=t^{-1} are two solutions of the differential equation

for equation (0.9) for $t > 0$.

(a) Verify that $y_1(t)$=t^2 and $y_2(t)$=t^{-1} are solutions of the equation (0.9) for $t > 0$.

(b) Verify that $y_1(t)$ and $y_2(t)$ given above are linearly independent by checking their Wronskian.

(c) Find the solution for equation (0.9) that satisfies the initial condition (0.10).

(d) Find the longest interval in which the given initial value problem is certain to

have a unique twice differentiable solution.

6. (almost 2.3 #4) A tank with a capacity of 500 gallon originally contains 100 gal of water with 100 lb of salt

in solution. Water containing 1 1b of salt per gallon is entering at a rate of 3 gal/min, and the mixture

is allowed to flow out of the tank at a rate of 2 gal/min.

(a) When does the solution begin to overflow? Why?

(b) Find the amount of salt in the tank at any time prior to the instant when the
solution begins to overflow.

7. (almost Section 3.6 #4) Find the solution for the initial value problem:

$$y'' + 2y' = 3 + 4\sin 2t, \tag{0.11}$$
$$y(0) = 1, \qquad y'(0) = 0. \tag{0.12}$$

8. (3.6 #15) Find the solution for the initial value problem:

$$y'' - 2y' + y = 4 + te^t, \tag{0.13}$$
$$y(0) = 1, \qquad y'(0) = 1. \tag{0.14}$$

9. Find the solution for the initial value problem:

$$y'' + 9y = 3\cos 3t + 4e^{2t}, \tag{0.15}$$
$$y(0) = 1, \qquad y'(0) = 0. \tag{0.16}$$

10. (almost 5.1 #8) Find the radius of convergence for the power series

$$\Sigma_{n=1}^{\infty} \frac{n^n}{n!} x^n$$

11. Find the radius of convergence for the power series

$$\Sigma_{n=0}^{\infty} \frac{n^2 + 2^n}{2^n n^2} x^n$$

12. Consider the infinite series for $0 \leq x < 1$:

$$S(x) = 1 + x + x^{2!} + x^{3!} + \cdots + x^{n!} + \cdots, \qquad 0 \leq x < 1 \tag{0.17}$$

(a) Show that the partial sum sequence $\{S_n(x)\}$ defined by the above infinite series is increasing.

(b) Show that the partial sum $S_n(x)$ is bounded above. Thus $\lim_{n\to\infty} S_n(x)=S(x)$.

(c) Use the Ratio Test to verify that the series $S(x)$ is convergent for $0 \leq x < 1$.

13. Consider the function

$$f(x) = \sqrt{1-x} \tag{0.18}$$

(a) Find its power series expansion at x=0 in the general terms.

(b) Find the interval of convergence for the above power series.

14. (Section 5.2 #10) Solve the differential equation by using the power series in the form
$\Sigma_{n=0}^{\infty} a_n x^n$ about the origin x_0=0:

$$(4-x^2)y'' + 2y = 0, \tag{0.19}$$

$$y(0) = 1, \qquad y'(0) = 0. \tag{0.20}$$

(a) Find the recurrence relation for the coefficients a_n.

(b) Given the initial conditions above, what are values for a_0 and a_1?

(c) Find the first four terms of the solution, i.e. for n=0, 1, 2, 3.

15. (a) (Section 7.3 #16) Find the eigenvalues and eigenvectors for the following matrix:

$$\begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$$

(b) (Section 7.3 #22) Find the eigenvalues and eigenvectors for the following matrix:

$$\begin{pmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{pmatrix}$$

16. (a) (Section 7.5 #12) Find the general solution for the system

$$\mathbf{x}' = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} \mathbf{x} \tag{0.21}$$

(b) Determine the particular solution for given initial condition:

$$\mathbf{x}(0) = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

17. (Section 7.6 #8)Find the general solution and a fundamental matrix for

$$\mathbf{x}' = \begin{pmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ -2 & -1 & 0 \end{pmatrix} \mathbf{x} \tag{0.22}$$

18. (Section 7.9 #8) Find the general solution of the given system of equations:

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^t \tag{0.23}$$