7. This problem can certainly be done by variation of parameters. However, since I mentioned that one can also reduce problems of this type to scalar second order equations and then solve them by the method of Section 3.6, I will carry that out here.

We are given the pair of equations

$$x_1' = x_1 + x_2 + 2e^t \text{ and } x_2' = 4x_1 + x_2 - e^t.$$

Solving the first equation for x_2 and substituting the result into the second equation, gives

$$x_2 = x_1' - x_1 - 2e^t \text{ and } x_1'' - x_1' - 2e^t = 4x_1 + x_1' - x_1 - 2e^t - e^t.$$

Simplifying the last equation, gives

$$x_1'' - 2x_1' - 3x_1 = -e^t.$$

This has the characteristic polynomial $r^2 - 2r - 3 = (r+1)(r-3)$. So trying $x_1 = Ae^t$ one finds $x_1 = (1/4)e^t$ is a particular solution, and

$$x_1 = \frac{1}{4}e^t + c_1e^{-t} + c_2e^{3t}$$

is the general solution. Then we have

$$x_2 = x_1' - x_1 - 2e^t = -2c_1e^{-t} + 2c_2e^{3t} - 2e^t.$$

So the general solution to the system is

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = e^t \begin{pmatrix} \frac{1}{4} \\ -2 \end{pmatrix} + c_1e^{-t} \begin{pmatrix} 1 \\ -2 \end{pmatrix} + c_2e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$