13. Furnish the details in determining when the steady-state response given by Eq. (11) is maximum; that is show that ω_{max} and R_{max} are given by Eqs. (12) and (13), respectively

Solution. This is just asking you to show that the amplitude of the steady state response in (11)

$$R(\omega) = \frac{F_0}{\sqrt{m^2(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2}}$$

really does assume the maximum value given in (13) at the value of ω given in (12). There is a way to save some computation here: $R(\omega)$ is going to assume its maximum when

$$\sqrt{m^2(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2}$$

assumes its minimum, and the square of a nonnegative function assumes its minimum where the function assumes its minimum. So all you have to do is find the minimum of

$$f(\omega) = m^2(\omega^2 - \omega_0^2)^2 + \gamma^2\omega^2$$

Since

$$f'(\omega) = 2m^2(\omega^2 - \omega_0^2)(2\omega) + 2\gamma^2\omega = 2\omega(2m^2(\omega^2 - \omega_0^2) + \gamma^2),$$

solving $f'(\omega) = 0$ gives

$$\omega_{\text{max}}^2 = \omega_0^2 - \frac{\gamma^2}{2m^2},$$

and substituting this into the formula for $R(\omega)$ gives

$$R_{\text{max}} = R(\omega_{\text{max}} = \frac{F_0}{\sqrt{\frac{\gamma^4}{4m^2} + \gamma^2(\omega_0^2 - \frac{\gamma^2}{2m^2})}} = \frac{F_0}{\gamma}\frac{1}{\sqrt{\omega_0^2 - \gamma^2/4m^2}}$$

which is the same as (13).