NAME

Quiz VII

1. (6 pts) Find the solution to

$$\underline{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \underline{x},$$

satisfying

$$\underline{x}(0) = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

Solution. Finding eigenvalues:

$$\det \begin{pmatrix} 1-r & 1 \\ 4 & -2-r \end{pmatrix} = r^2 + r - 6 = (r+3)(r-2)$$

So eigenvalues are 2 and -3. Finding eigenvector for $r = 2$:

$$\begin{pmatrix} -1 & 1 \\ 4 & -4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ and an obvious solution is } \underline{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Finding eigenvector for $r = -3$:

$$\begin{pmatrix} 4 & 1 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ and an obvious solution is } \underline{v} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

So the general solution is

$$\underline{x}(t) = c_1 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-3t} \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

To satisfy the initial condition you need

$$\begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Solving that you easily find

Ans.

$$\underline{x}(t) = e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} - e^{-3t} \begin{pmatrix} 1 \\ -4 \end{pmatrix}.$$