NAME

Quiz IV

1. (3 pts) Determine the set of x for which the series

$$S(x) = \frac{2}{x^2} + \frac{4}{x^4} + \cdots + \frac{2^n}{x^{2n}} + \cdots$$

converges, and compute $S(x)$ for those x.

Solution. This is the series

$$y + y^2 + \cdots + y^n + \cdots$$

with $y = 2/x^2$. Our basic theorem on geometric series says it converges when $|y| < 1$, and, since the first term in the geometric series – the 1 – is missing, the sum will be

$$\frac{1}{1-y} - 1 = \frac{y}{1-y}. \qquad (*)$$

When is $|y| = 2/x^2$ less than one? Since $2/x^2 < 1$ is equivalent to $x^2 > 2$, $2/x^2 < 1$ precisely when $|x| > \sqrt{2}$. Substituting $y = 2/x^2$ into (*) gives the answer.

Ans. For $|x| > 2$

$$S(x) = \frac{2/x^2}{1 - 2/x^2} = \frac{2}{x^2 - 2}.$$

2. (3 pts) Find the largest interval $-R < x < R$ on which

$$\sum_{n=0}^{\infty} ne^{-n}x^{2n}$$

converges.

Solution. This is just the ratio test:

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{(n+1)e^{-n-1}|x|^{2n+2}}{ne^{-n}|x|^{2n}} = \lim_{n\to\infty} \frac{n+1}{n}e^{-1}|x|^2$$

$$= \lim_{n\to\infty} (1 + \frac{1}{n})e^{-1}|x|^2 = e^{-1}|x|^2$$

So the ratio test says this converges for $e^{-1}|x|^2 < 1$ and diverges for $e^{-1}|x|^2 > 1$. This determines the interval of convergence.

Ans. The interval of convergence is $-e^{1/2} < x < e^{1/2}$.