NAME

Quiz III

1. (3 pts) Find the solution of $y'' - 2y' + y = 2t + 1$ which satisfies $y(0) = 3$ and $y'(0) = 3$.

Solution. Using the method of undertermined coefficients, set $y = At + B$. Substituting this into the equation

$$(At + B)'' - 2(At + B)' + (At + B) = 0 - 2A + (At + B) = 2t + 1.$$

So $A = 2$ and $B = 5$. The general solution to the homogeneous equation is $y = e^t(c_1 t + c_2)$, so the general solution to $y'' - 2y' + y = 2t + 1$ is

$$y = 2t + 5 + e^t(c_1 t + c_2)$$

Since $y(0) = 5 + c_2$ and $y'(0) = 2 + c_1 + c_2$, we have $c_2 = -2$ and $c_1 = 3$.

Ans. $y = 2t + 5 + e^t(3t - 2)$.

2. (3 pts) Find a particular solution to

$$y'' - 9y = g(x).$$

Here $g(x)$ is supposed to be a general continuous function.

Solution. Two linearly independent solutions to the homogeneous equation $y'' - 9y = 0$ are $y_1 = e^{3x}$ and $y_2 = e^{-3x}$. For this problem we have to use variation of parameters because g does not have one of the special forms needed for undetermined coefficients. So we need to solve

$$c_1' y_1 + c_2' y_2 = 0$$

$$c_1' y_1' + c_2' y_2' = g.$$

That is

$$c_1' e^{3x} + c_2' e^{-3x} = 0$$

$$3c_1' e^{3x} + (-3) c_2' e^{-3x} = g$$

So

$$c_1' = \frac{1}{6} e^{-3x} g(x) \text{ and } c_2' = \frac{-1}{6} e^{3x} g(x).$$

Ans

$$y = e^{3x} \int \frac{1}{6} e^{-3x} g(x) dx + e^{-3x} \int \frac{-1}{6} e^{3x} g(x) dx$$