

4. A tank with capacity 500 gal originally contains 200 gal water with 100 lb salt in solution. Water containing 1 lb salt per gallon enters tank at rate 3 gal per minute. The mixture flows out of the tank at a rate of 2 gal per minute. Find the concentration (lb per gal) of salt in the tank just before it overflows. [The rest of the problem asked what the limit concentration would be if the tank were infinitely large – but pretty obviously that is going to be the concentration in the mixture flowing in, 1 lb per gal.]

Solution: Note that the amount of liquid in the tank is increasing at a rate of 1 gal per minute – 3 gal per minute is flowing in and only 2 gal per minute is flowing out. So there will be  $200+t$  gal in the tank at time  $t$ , and the tank will overflow when  $t=300$ . Let  $S(t)$  be the number of lbs of salt in the tank at time  $t$ . The rate of change of  $S(t)$  is

[rate salt in – rate salt out

So

$$\frac{dS}{dt} = (1\text{lb/gal})(3\text{gal/min}) - (S(t)/(200 + t))(2\text{lb/min})$$

or suppressing the units and writing the equation in standard form

$$\frac{dS}{dt} + \frac{2S}{200 + t} = 3$$

To solve this you multiply by  $e^A$  where  $dA/dt = 2/(200 + t)$ , but it is important to notice that in this case  $e^A$  is just  $(200 + t)^2$ . So

$$\frac{d}{dt}((200 + t)^2 S) = 3(200 + t)^2, \text{ and}$$

$$S = (200 + t) + C(200 + t)^{-2}. \text{ Since } S(0) = 100, C = -(100)(200)^2.$$

Thus  $S(300) = 500 - (100)(200)^2/(500)^2$  and the concentration,

$$C = \frac{S(300)}{500} = 1 - \frac{4}{125}.$$