

The Bootstrap Part of 1.22

Suppose $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is locally Lipschitz, and set

$$M(r) = \sup\{\|F(u)\|, \|u\| < r\}.$$

Show that $\dot{u} = F(u)$, $u(0) = u_0$ has a solution on the interval $|t| \leq \delta$, where $\delta > 0$ depends only on $\|u_0\|$ and $M(r)$. In particular it does not depend on the local Lipschitz constants of F .