There is more than one way to skin a cat, as people used to say. Instead of following the suggestions here one could start with the case where $\alpha > 0$ and β is real. In that case it is easy to reduce the integral to $\int_{\mathbb{R}} e^{-x^2} dx$. Almost everyone has heard about the trick for doing that one (square it, interpret the result as an integral over $\mathbb{R}^2$, and evaluate that using polar coordinates). Once you get the formula for β real and $\alpha > 0$ consider analytic continuation on both sides of the equation, first in β to all of $\mathbb{C}$, and then in α to $\mathrm{Re}\{\alpha\} > 0$ – and even to $\mathrm{Re}\{\alpha\} \geq 0$ to get the formula for the fundamental solution for the initial value problem for the Schrödinger equation.