NAME

Truth or Counterexamples

Part A. Decide whether the following statements are true or false. Mark the true ones "T" and the false ones "F". All sets and funtions are measurable.

a) If, for each n, f_n is continuous on [0,1] and $\lim_{n\to\infty} f_n(x) = 0$ for each $x \in [0,1]$, then $\lim_{n\to\infty}(\sup_{x\in[0,1]} f_n(x)) = 0$.

b) If, for each n, you have $|f_n(x) - f(x)| < 1/n$ for all $x \in \mathbb{R}$ and the f_n's are integrable, then $\lim_{n\to\infty} \int_{\mathbb{R}} f_n dx = \int_{\mathbb{R}} f dx$.

c) If $f_n(x) \geq 0$ and $\lim_{n\to\infty} f_n(x) = f(x)$ for all x, then $f \notin L^1$ implies $\lim_{n\to\infty} \int f_n dx = \infty$.

d) If $E = \cap_{n=1}^{\infty} F_n$ and $m(F_1) < \infty$, then $\liminf m(F_n) = m(E)$.

e) If $\int_0^1 |f_n| dx \to 0$ as $n \to \infty$ and $f_n \in C_c(\mathbb{R})$, then there is an N such that $|f_n(x)| \leq 1$ for all $x \in [0,1]$ when $n > N$.

f) If $f_n(x) \geq f_{n+1}(x) \geq 0$ and $\int_{\mathbb{R}^d} f_1 dx < \infty$, then $\lim_{n\to\infty} \int_{\mathbb{R}^d} f_n(x) dx = \int_{\mathbb{R}^d} \lim_{n\to\infty} f_n(x) dx$.

g) If $||f_n - f||_{L^1(\mathbb{R}^d)} \to 0$ as $n \to \infty$ and $\int_{\mathbb{R}^d} f_n dx = 1$ for all n, then $\int_{\mathbb{R}^d} f dx = 1$.

h) If $f \in L^1(\mathbb{R}^d)$, then $\lim_{n\to\infty} m(\{x : f(x) > n\}) = 0$.

Part B. Give a counterexample to each statement you said was false in Part A. You do not need to prove that your counterexample works, but state clearly what it **is**.