## Assignment 7

**1.** (Royden) Suppose that $f$ is absolutely continuous on $[\epsilon, 1]$ for each $\epsilon > 0$ and continuous on $[0, 1]$. Does it follow that $f$ is absolutely continuous on $[0, 1]$? What if $f \in BV[0, 1]$?

**2.** (Rudin, sort of) Assume $0 \leq f_1(x) \leq f_2(x) \leq \cdots \leq 1$ and let $f(x) = \lim_{n\to\infty} f_n(x)$. If each $f_n$ is nondecreasing, is

$$\lim_{n\to\infty} f_n'(x) = f'(x) \ a.e.?$$

What if the $f_n$'s are also absolutely continuous?

3. (Rudin) Find an example of a positive continuous function on the open unit square, $(0, 1) \times (0, 1)$, in $\mathbb{R}^2$, whose integral is finite, but

$$\phi(x) = \int_0^1 f(x, y)dy$$

is infinite for some $x \in (0, 1)$.

Also from the exercises for Chapter 3 in Stein & Shakarchi Exercises **22** and **25**.