Assignment 6

1. (Quals) Suppose that $f(x) = \sum_{n=0}^{\infty} a_n x^n$, where the a_n's are real and

$$\sum_{n=0}^{\infty} |a_n| < \infty.$$

Show that f is of bounded variation on $[-1, 1]$. Hint: Treat the case where all of the a_n's are nonnegative first.

2. (Quals) Let f be a monotone nondecreasing function on $[0, 1]$ with $f(0) = 0$.
   a) What can be said about the derivative of f?
   b) What can be said about $F(x) = \int_0^x f'(t)dt$?
   c) What can be said about $F'(x)$?
   d) What can be said about $G(x) = \int_0^x F'(t)dx$?

["What can be said about..." is a rather vague question. You should interpret it as "What do the theorems we have learned say about..."]

From the Exercises for Chapter 3 (pp 145-152): Exercises 7, 9, 20(a) and 23. In problem 9 the notation $\delta(x + y) = o(|y|)$ means $\lim_{|y|\to 0} \delta(x + y)/|y| = 0$.