Assignment 4

1. (from Real Analysis Qual from long ago) Evaluate

$$\lim_{N\to\infty} N \int_0^N \frac{1}{s} \log(1 + (s/N)) \frac{ds}{1+s^2}.$$

Justify your calculations.

2. (likewise from an ancient qual) Let $\{f_n\}$ be a sequence in $L^1(\mathbb{R})$ such that

$$\int_{-\infty}^{\infty} |f_n(t) - f(t)| dt \leq n^{-2} \ (n \geq 1).$$

Show that f_n converges pointwise to f a.e.

These are from Stein and Shakarchi, pp. 89-97: Exercises 4 and 16, and Problem 2. Note that Problem 2 comes immediately after Problem 1. To see why Cantor and Lebesgue might have been interested in this question note that

$$\sum_{n=1}^{\infty} \sin(\pi n! x)$$

converges for all rational numbers x.