PROBABILITY

Counting

Multiple Stages

With replacement

- HOW MANY PASSWORD ARE POSSIBLE STARTING WITH TWO LETTERS FOLLOWED BY FOUR NUMERALS?

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- $(26)(26)(10)(10)(10)(10) = 6{,}760{,}000$

WITHOUT REPLACEMENT

– How many three letter words from { a,b,c,d,e} without using a letter more than once?

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- Ans. (5)(4)(3) = 60

With restrictions

- How many three letter words from { a,b,c,d,e} without using a letter more than once?

- How many three letter words from { a,b,c,d,e} without using a letter more than once and a vowel in the middle?
- Ans. (4)(2)(3) = 24

Constructive counting (Cases)

- How many three digit numbers have one zero?

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- Case 1 zero in the middle (9)(1)(9) = 81
- Case 2 zero on the right (9)(9)(1) = 81
- Total 81 +81 =162

Complementary (finding opposite first)

- How many ways to seat four boys and three girls in a row so that at least two boys sit together?

- How many ways to seat four boys and three girls in a row so that at least two boys sit together?

- Ans. Only one way for two boys <u>not</u> to sit together: BGBGBGB. There are 4! ways to arrange the boys and 3! Ways to arrange the girls. (4!)(3!) = (24)(6) = 144. There are 7! = 5040 ways to seat seven. Thus 5040-144 = 4896 ways.

Multiple methods

- In how many ways may 3 officers (Pres. ,V.P. , Sec) be picked from a 12 member club, If Al and Bob won't serve together?

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- Ans. Ways to pick three of 12: (12)(11)(10) =1320
- Ways to pick three with Al and Bob: (3)(2)(10) =60. Thus 1320-60 = 1260 ways to pick without both.

Overcounting then correcting

- How many ways tho arrange the letters of Mississippi?

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- Ans. There are 11! ways to arrange the letters. There are 4! ways to arrange the “s” and “I”, and 2! For “p”. Thus 11!/[(4!)(4!)(2!) =34650 diferent arrangements.

Symmetry

- How many ways can 6 people be seated at a round table. It does not matter who is seated north . What matters is who is on a persons left and right.

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- Ans. Take one person (say A) seat her at a definite place (say north). This leaves 5! Ways to seat the others. Thus there are (n-1)! Ways to seat n at a round table.

Combinations
(order does not matter)

– In How many ways can a committee of four be picked from a club of ten.

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- Ans. There are (10)(9)(8)(7) way to pick, but there are 4! Ways to arrange each pick so there are duplications. Thus there are (10)(9)(8)(7)/4! Different committees. Combinations are represented by ${}_{10}C_4$.

Probability

- If all outcomes equally likely.
  P(success) = (# of successful outcomes)/(# of possible outcomes).

- e.g. Probability of drawing a face card from a deck of playing cards : 12 face cards/ (52 cards) = 3/13.

Multistep problem

- Find the probability of drawing a full house (three of one rank and two of another rank) in receiving 5 cards from a deck of 52 cards.

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- Ans. Method one:
- Count of successes divided by count of possib
- Successes : For the three of a kind there are 1 and $_4C_3$ ways of picking three. (13)(4)= 52 su For two of a kind 12 ranks and $_4C_2$. (12)(6) =72 successes: (52)(72)
- Possibilities: $_{52}C_5$.
- Prob.= (52)(72)/[(52)(51)(50)(49)(48) /5!] = 6/4

- Method two:
- <u>Product of possibilities</u>
- Find probability of <u>one</u> successful order.
- Then consider arrangements.
- Pick three alike followed by two alike
- (52/52)(3/51)(2/50)(48/49)(3/48) = 3/20825
- 5! /[(3!)(2!)] = 20 arrangements
- Prob. = (20)(3)/20825 = 6/4165

Using Algebra

- A bag has 3 red and k marbles.
- The probability is ½ that 2 marbles chosen at random will both be the same color. Find all values of k.

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- The probability is ½ that 2 marbles chosen at random will both be the same color. Find all values of k.
- Ans.
- Case 1: both red
- $[3/(3+k)][2/(2+k)] = 6/(k^2+5k+6)$
- Case 2: both white
- $[k/(3+k)][(k-1)/(2+k)] = (k^2-k)/(k^2+5k+6)$
- Sum $(k^2-k+6)/(k^2+5k+6) = ½$ Solution $k=1,6$

Complementary

- If the probability of catching a fish is 1/3 each day. What is the least number of days you need to fish to have at least a 90% chance of catching a fish?

- If the probability of catching a fish is 1/3 each day. What is the least number of days you need to fish to have at least a 90% chance of catching a fish?
- Ans. You have 2/3 chance not to catch a fish each day. Find n such that $(2/3)^n < 0.1$
- n = 5 $1- (2/3)^5 = 311/343 = .9067$

- Cal and Dan each sit randomly in a row of 7 chairs. What is the probability that they don’t sit together?

- Cal and Dan each sit randomly in a row of 7 chairs. What is the probability that they don't sit together?
- Ans. ${}_7C_2$ = 21 ways they can pick two chairs. There are 6 ways they can sit together ; (1,2) (2,3) (3,4) … (6,7).
- Probability of sitting together. 6/21 = 2/7.
- Probability of not sitting together 1-2/7 = 5/7

Geometry

- A point is randomly picked in a 4cm X 4cm square. A circle of 1 cm radius is draw with the point as center. What is the probability that the circle is entirely within the square?

- A point is randomly picked in a 4cm X 4cm square. A circle of 1 cm radius is draw with the point as center. What is the probability that the circle is entirely within the square?
- ANS. Consider lines in the square 1cm from each side. This forms a 2cm X 2cm square which is the loci for the center of the circles that are within the larger square. Thus the probability is the ratio of the areas. 4/16 = 1/4

Expected return

- A dart board consists of of concentric circles of radii 1cm, 2cm, and 3cm. If the dart lands in the inner circle you get 25 cents, between the 1^{st} and 2^{nd} 10 cents, and between the 2^{nd} and 3^{rd} 5 cents. If the dart lands randomly on the board, what is your expected return?

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- Ans. The 25 cent area is 1π, the 10 cent $(4-1)\pi$, and the 5 cent area is $(9-4)\pi$. The expected return is $25(1/9) + 10(3/9) + 5(5/9) = 80/9$ cents.

Binomial expansion

- If p is the probability of an event occurring during a single trial and q is the probability of it not occurring (i.e. 1-p), then $(p + q)^n$ can be used to find the probability of any number of p's from 0 to n occurring in n trials.
- For example: if a spinner lands on A one third of the time, What is the probability of getting four A's in five spins?

- For example: if a spinner lands on A one third of the time, What is the probability of getting four A’s in five spins?
- Ans. $(1/3 + 2/3)^5$ yields ${}_5C_5(1/3)^5(2/3)^0$ for 5 p’s and ${}_5C_4(1/3)^4(2/3)^1 = 10/243$ for 4 p’s.

- 11. A box contains exactly five chips, three red and two white. Chips are randomly
- removed one at a time without replacement until all the red chips are drawn or
- all the white chips are drawn. What is the probability that the last chip drawn
- is white?
- (A) 3/10
- (B) 2/5
- (C) 1/2
- (D) 3/5
- (E)7/10

- 16. Tina randomly selects two distinct numbers from the set {1, 2, 3, 4, 5}, and Sergio
- randomly selects a number from the set {1, 2, . . . , 10}. The probability that
- Sergio’s number is larger than the sum of the two numbers chosen by Tina is
- (A) 2/5 (B) 9/20 (C) 1/2 (D) 11/20 (E) 24/25

- 22. Triangle ABC is a right triangle with angle ACB as its right angle, m angle ABC = 60○,
- and AB = 10. Let P be randomly chosen inside △ ABC, and extend BP to
- meet AC at D. What is the probability that BD > $5\sqrt{2}$?

- (A) $(2 - \sqrt{2})/2$
- (B) 1/3
- (C) $(3 - \sqrt{3})/3$
- (D) 1/2
- (E) $(5 - \sqrt{5})/5$

- 18. A point P is randomly selected from the rectangular region with vertices (0, 0), (2, 0),
- (2, 1), (0, 1). What is the probability that P is closer to the origin than it is to
- the point (3, 1)?
- (A) 1/2
- (B) 2/3
- (C) 3/4
- (D)4/5
- (E) 1

- 15. Brenda and Sally run in opposite directions on a circular track, starting at
- diametrically opposite points. They rst meet after Brenda has run 100 meters.
- They next meet after Sally has run 150 meters past their rst meeting point.
- Each girl runs at a constant speed. What is the length of the track in meters?
- (A) 250 (B) 300 (C) 350 (D) 400 (E) 500

- 20. Select numbers a and b between 0 and 1 independently and at random, and let
- c be their sum. Let A, B, and C be the results when a, b, and c, respectively,
- are rounded to the nearest integer. What is the probability that A + B = C?
- (A) 1/4
- (B) 1/3
- (C) 1/2
- (D) 2/3
- (E) 3/4

- 22. Three mutually tangent spheres of radius 1 rest on a horizontal plane. A sphere
- of radius 2 rests on them. What is the distance from the plane to the top of the
- larger sphere?
- (A) $3 + \sqrt{30}/2$
- (B) $3 + \sqrt{69}/3$
- (C) $3 + \sqrt{123}/4$
- (D) $52/9$
- (E) $3 + 2\sqrt{2}$

- 24. A plane contains points A and B with AB = 1. Let S be the union of all disks of radius 1 in the plane that cover AB. What is the area of S?
- (A) $2\pi + \sqrt{3}$
- (B) $8/3\pi$
- (C) $3\pi - \sqrt{3}/2$
- (D) $10\pi - \sqrt{3}$

(E) $4\pi - 2\sqrt{3}$

- 25. For each integer $n \geq 4$, let a_n denote the base-n number 0:133n. The product
- $a_4 a_5 : : : a_{99}$ can be expressed as m/n! , where m and n are positive integers and n
- is as small as possible. What is the value of m?
- (A) 98 (B) 101 (C) 132 (D) 798 (E) 962