LAMC – 27th of April, 2008

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1. You are given a conventional deck of 52 cards, well-shuffled. What is the probability that the top card is an ace?

2. Again you are given a deck of cards, you count the cards (face down) and find it defective, having only 51 cards. What is the probability that the top card is an ace?

3. Monty Hall Suppose yourself to be participating in a bizarre show contest. You have a choice of three doors. Behind one door is a costly car, behind the other two doors are cheap goats. The host, who knows where the car and goats are promises you, that once you have chosen a door, he will open a door with a goat behind it and will offer you to switch doors. You choose the first door, whereupon the host of the show opens the third door to reveal a goat. He then offers you to change your choice of door. Would you?

4. You flip three fair coins. at least two are alike and it is an even chance that the third is head or tail, so the chance that all three are the same is $\frac{1}{2}$, right?

5. A test for a virus shows a positive result in 99% of all cases when the virus is actually present and in 5% of all cases when the virus is not present (a *false positive* result). Assume that the prevalence of the virus in the population is 1 in 200.

   (a) If such a test is administered to a randomly chosen individual, what is the probability that the rest result is positive?

   (b) Given that the test came out positive, what is the probability that the individual is actually infected by the virus?

   (c) Given that the test came out negative, what is the probability that the individual is not infected?

6. Genetics Mutation Assume that a certain disease is caused by a genetic mutation or appears spontaneously. The disease will appear in 60% of all people with the mutation, and in 20% of all people without the mutation. Assume that four percent of the population has the genetic mutation.

   (a) What is the probability that a randomly chosen individual has the disease?

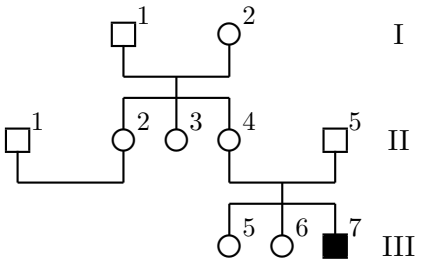

Figure 1: Pedigree 1

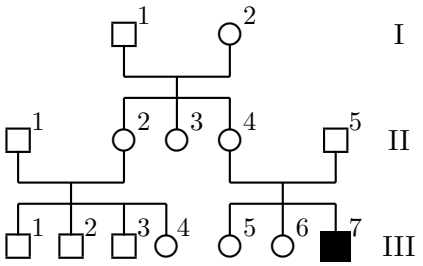

Figure 2: Pedigree 2

(b) Given that a randomly chosen individual suffers from the disease, what is the probability that the individual has the genetic mutation?

7. Disease A certain disease is twice as likely to develop when the person is a smoker as it is when (s)he is a nonsmoker. If 22% of the people are smokers, what percentage of people having this disease are smokers?

8. Hemophilia[1]

   (a) Given pedigree 1, find the probability that II-2 is a carrier. Show work and justify your answer.

   (b) Given pedigree 2, find the probability that II-2 is a carrier. Show work and justify your steps.

9. Murder A murder is committed. the murder is either one of the other of two persons X and Y. Both persons are on the run from the authorities, and, after an initial investigation, both fugitives appear to be equally likely to be the perpetrator. Further investigation reveals the the perpetrator has blood type A. 10% of the population has blood type A. Additional inquiry reveals, that person X has blood type A, but offers

[1]Hemophilia is a blood disorder that is characterized by a blood-clotting factor deficiency. Individuals afflicted with this disease suffer from excessive bleeding. The disease is caused by an abnormal gene that resides on the X-chromosome. A female who carries the abnormal gene on one of her X-chromosomes but not on the other is a carrier of the disease but will not develop symptoms. A male who carries the abnormal gene on his (only) X-chromosome will develop symptoms of the disease.

If the father carries the abnormal gene (and thus suffers from hemophilia), all his daughters will be carriers since they inherit their father's X-chromosome; but all his sons will be disease free since they inherit their father's Y-chromosome. If the mother carries the abnormal gene, then her daughters have a 50% chance of being carriers and her sons have a 50% chance of suffering from the disease.

In a pedigree, males are denoted by squares, females by circles; blackened symbols denote individuals that suffer from the disease that is tracked by the pedigree.

From Claudia Neuhauser: Calculus for Biology and Medicine, Second Edition, pages 815-816.

no information concerning the blood type of person Y. In light of this new information, what is the probability that person X is the perpetrator?

10. Coins

A man has five coins, two of which are double-headed, one is double-tailed, and two are normal.

(a) The man closes his eyes, picks one coin at random, and tosses it. What is the probability that the lower face of the coin is a head?

(b) He opens his eyes, looks at the coin and sees that the upper face is a head. What is the probability now that the lower face is a head?

(c) He closes his eyes again, and tosses the (same) coin again. What is the probability now that the lower face is a head?

(d) He opens his eyes, looks at the coin and sees that the upper face is a head, again. What is the probability now that the lower face is a head?