

DUE 9/23/07

- Please, submit your solutions to different problems on separate pieces of paper, together with your name, grade and school on each sheet;
- You can consult any resources (books, notes, internet), but you may not receive help from other people.

Problem 1. Every day at noon, a ship starts sailing from port A to port B , and at the same time a ship starts from B to A . A trip in either direction takes exactly 7 days. How many ships, sailing in the opposite direction, will the ship sailing today from A meet on its trip?

Problem 2. Compute the (exact value of the) sum

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \cdots + \frac{1}{\sqrt{9999} + \sqrt{10000}}.$$

Problem 3. Without using any electronic computational devices (such as calculators, computers, etc.), determine which of the numbers is greater: 2^{300} or 3^{200} ?

Problem 4. 20 points are placed around a circle. Players take turns joining two of the points with a line segment which does not cross a segment already drawn in. The player who can not do so loses. Which player (first or second) can win this game and how?

Problem 5. The famous Pythagorean triple of numbers 3, 4, 5 has the property that

- the numbers are consecutive;
- the square of the third equals to the sum of the squares of the first two:

(1)
$$3^2 + 4^2 = 5^2.$$

Prove that there is no other triple of integers (i.e., positive whole numbers) with the same property (that the three numbers are consecutive and the squares of the two smaller ones add up to the square of the biggest).