When the bases are powers of each other, we can convert numbers from one base to another by breaking them apart.

**Example 1.** What is $312_4$ in base 2?

1. Expand $312_4$ by writing it in terms of powers of 4:
   
   $312_4 = ____ \times 4^2 + ____ \times 4^1 + ____ \times 4^0$

2. Write each term from the expression above in base 2:
   
   (a) $____ \times 4^2$
      
      = ________ (in base 10)
      
      $= ____ \times 2^5 + ____ \times 2^4 + ____ \times 2^3 + ____ \times 2^2 + ____ \times 2^1 + ____ \times 2^0$

   (b) $____ \times 4^1$
      
      = ________ (in base 10)
      
      $= ____ \times 2^3 + ____ \times 2^2 + ____ \times 2^1 + ____ \times 2^0$

   (c) $____ \times 4^0$
      
      = ________ (in base 10)
      
      $= ____ \times 2^1 + ____ \times 2^0$
(3) Add all of the terms above to get your final answer:

312₄ = ___ × 4² + ___ × 4¹ + ___ × 4⁰

= ___ × 2⁵ + ___ × 2⁴ + ___ × 2³ + ___ × 2² + ___ × 2¹ + ___ × 2⁰

= ________ (in base 2)
Example 2. What is $73_8$ in base 2?

(1) Expand $73_8$ by writing it in terms of powers of 2:

$$73_8 = \_\_ \times 8^1 + \_\_ \times 8^0$$

(2) Write each term from the expression above in base 2:

(a) $\_\_ \times 8^1$

$$= \_\_ \text{ (in base 10)}$$

$$= \_\_ \times 2^5 + \_\_ \times 2^4 + \_\_ \times 2^3 + \_\_ \times 2^2 + \_\_ \times 2^1 + \_\_ \times 2^0$$

(b) $\_\_ \times 8^0$

$$= \_\_ \text{ (in base 10)}$$

$$= \_\_ \times 2^2 + \_\_ \times 2^1 + \_\_ \times 2^0$$

(3) Add all of the terms above to get your final answer:

$$73_8 = \_\_ \times 8^1 + \_\_ \times 8^0$$

$$= \_\_ \times 2^5 + \_\_ \times 2^4 + \_\_ \times 2^3 + \_\_ \times 2^2 + \_\_ \times 2^1 + \_\_ \times 2^0$$

$$= \_\_ \text{ (in base 2)}$$
(1) Using the method above, convert $21_4$ to base 2.

(2) Using the method above, convert $27_{18}$ to base 2.

(3) Using the method above, convert $68_9$ to base 3.
(4) Suppose you are given a number written in base 4. Describe in your own words how you would get the digits in base 2 from the digits written in base 4. Do you notice any patterns that would make the process faster?

(5) Can you use this pattern to quickly convert $BAD_{16}$ to base 2?

(6) How about $CAFE_{16}$ to base 4? (Hint: Remember that $16 = 4^2$)

(7) Can you use this pattern to do the opposite and convert $33000031_4$ to base 16? (Hint: Remember that $16 = 4^2$)
(8) In question (4), you described how to convert numbers from a higher base to a lower base that are powers of each other. Can we reverse the process you described in the question (4) above to convert from a lower base to a higher base? Suppose you are given a number written in base 2. Describe in your own words how you would get the digits in base 4 from the digits written in base 2.

(9) Can you use this method to convert a number written in base 125 to base 5? Why or why not? (Hint: Remember that $125 = 5^3$)

(10) Can you use this method to convert a number written in base 5 to base 2? Why or why not?
(1) Convert the following powers to the other:
(a) $4^1 = 2^-$

(b) $3^1 = 9^-$

(c) $5^1 = 25^-$

(d) $18^0 = 2^-$

(e) $2^3 = 8^-$

(f) $4^n = 2^-$

(g) $27^n = 3^-$

(h) $125^n = 5^-$

(i) $2^{600n} = 8^-$
When we convert numbers between base $n$ and $n^2$, we can use a similar method of breaking the numbers apart.

**Example 3.** What is $134_{n^2}$ in base $n$ where $n > 4$?

1) Expand $134_{n^2}$ by writing it in terms of $n^2$:

$$134_{n^2} = 1 \times (n^2)^{-1} + 3 \times (n^2)^{0} + 4 \times (n^2)^{-1}$$

$$= 1 \times n^{-1} + 3 \times n^{-0} + 4 \times n^{-1}$$

2) Write each term above into the form of terms in base $n$:

$$134_{n^2} = \_\_ \times n^4 + \_\_ \times n^3 + \_\_ \times n^2 + \_\_ \times n^1 + \_\_ \times n^0$$

3) Write your final answer in base $n$:

$$134_{n^2} = \_\_\_\_\_\_\_ \text{ (in base } n)$$

**Example 4.** What is $1BE_{n^2}$ in base $n$ where $n > 16$?

1) Expand $1BE_{n^2}$ by writing it in terms of $n^2$:

$$1BE_{n^2} = 1 \times (n^2)^{-1} + B \times (n^2)^{0} + E \times (n^2)^{-1}$$

$$= 1 \times n^{-1} + B \times n^{-0} + E \times n^{-1}$$

2) Write each term above into the form of terms in base $n$:

$$1BE_{n^2} = \_\_ \times n^4 + \_\_ \times n^3 + \_\_ \times n^2 + \_\_ \times n^1 + \_\_ \times n^0$$

3) Write your final answer in base $n$:

$$1BE_{n^2} = \_\_\_\_\_\_\_ \text{ (in base } n)$$
(1) Do you notice any patterns with your answers when converting from base $n^2$ to base $n$?

(2) Does this pattern still apply for converting a number like $890_{n^2}$ to base $n$ where $n \leq 9$?