**Algorithms**

It becomes complicated to find the shortest solutions without a clear set of orderly steps. We want to develop an algorithm, a set of precise steps to solve a problem.

**Wolf-Cabbage-Goat Problem**

A man has a cabbage, a goat, and a wolf. The animals are well trained, so in the presence of the man, everybody is safe. However, in the absence of the man, the goat will eat the cabbage, and the wolf will eat the goat. They all need to cross a river in a small boat that can only hold the man and one other item. How can the man get all his possessions across safely?
Let’s develop an algorithm to solve this problem. The first step is done for you. (The shore they’re coming from is called A, and the shore they’re going to is called B.)

Step 1: M+G, A→B
Step 2: M, B→A
Step 3: M+C, A→B
Step 4: M+G, B→A
Step 5: M+W, A→B
Step 6: M, B→A
Step 7: M+G, A→B

How many steps did it take? Can you find any shorter (optimal) way to do it?

7 steps
Hanoi Towers
May 3, 2015

Legends speak of a monastery where monks must move 64 golden disks from one rod to another. When all the disks are moved, the world will come to an end. We will play this game with fewer disks.

Purpose: Move all the disks from one stick to another.

Rules: 1) Only one disk can be moved at a time.

2) Larger disks cannot be placed on smaller disks.

1. If you have only two disks on rod A, draw all the steps you would make to move them to rod C. (Hint: Use rod B.)
2. When we have many disks it may be difficult to keep track of the moves. We will use the following notation for the moves: Number the disks 1 to N from the top to the bottom, and name the rods A, B, and C.

Ex. Moving disk 1 from rod A to rod B would be written as 1AB.

a) Write down the notation for the moves you made in the previous problem.
   
   (1) 1AB
   (2) 2AC
   (3) 1BC

b) Use the same notation to write down how you would move the two disks from rod A to rod B.

   (1) 1AC
   (2) 2AB
   (3) 1CB

3. Now you have three disks on the rod A. On the next page, use the smallest number of moves to move the tower from rod A to rod C. The first step is done for you.
Notation for the move.

(1) 1AC

(2) 2AB

(3) 1CB

(4) 3AC

(5) 1BA

(6) 2BC

(7) 1AC
a) How many steps did it take? 7

b) Write down the notation for each step next to the picture.

c) Describe in words what steps (1) – (3) do.

   *It opens up Ring C for Disk 3 to be placed in.*

d) Describe in words what step (4) does.

   *Put Disk 3 on the bottom of Ring C and begin building the final stack.*

e) Describe in words what steps (5) – (7) do.

   *Stacks up Disks 1 and 2 on top of Disk 3 on Ring C.*

f) What is the meaning of the notation 1,2 AB?

   *Moving Disks 1 and 2 from Ring A to Ring B.*

g) What is the meaning of the notation 3 AC?

   *Move Disk 3 from Ring A to Ring C.*

h) What is the meaning of the notation 1,2 BC?

   *Move Disks 1 and 2 from Ring B to Ring C.*
4. Find a partner and share a Hanoi Tower model. Place 4 disks on rod A. Your goal will be use the smallest number of moves to move the entire 4-disk tower to rod C following the rules of the game. For each step, use the notation to write down your moves.

Hint: First try to make the moves for 1, 2, 3 AB. You know that moving 3 disks takes 7 moves.

(1) 1AB
(2) 2AC
(3) 1BC
(4) 3AB
(5) 1CA
(6) 2CB
(7) 1AB
(8) 4AC
(9) 1BC
(10) 2BA
(11) 1CA
(12) 3BC
(13) 1AB
(14) 2AC
(15) 1BC
a) How many moves did it take to move the 4-disk tower from rod A to rod C?

15

b) Given that it takes 31 moves to move 5 disks from A to C, find how many moves it takes to move 6 disks from A to C. Hint: Break down the problem into smaller problems and solve the smaller problems first.

1 disk → 1 move
2 disks → 3 moves
3 disks → 7 moves
4 disks → 15 moves
5 disks → 31 moves

Break down the problem into these smaller problems:
Step 1) Transfer n-1 disks from A to B
Step 2) Transfer Disk 1 to C
Step 3) Transfer remaining from B to C

Find the pattern \((2^n-1)\)
6 disks → \(2^6-1=63\) moves