6 Nor gates

Another type of logic gate is called a “Nor gate”. A Nor gate has two inputs and one output, and is drawn on paper like this:

The output is “True” when neither Input 1 nor Input 2 is true. Otherwise, the output is “False”.

In this section, we will show that the Nor gate is also universal!

1. Fill in the missing inputs and outputs in the following Nor gates.

\[
\begin{array}{ccc}
T & \quad & F \\
T & \quad & ? \\
F & \quad & ? \\
T & \quad & ? \\
F & \quad & T \\
F & \quad & ? \\
F & \quad & F \\
\end{array}
\]
2. Draw a picture that shows how to make a Nor gate out of an Or gate and a Not gate:

\[ \text{Input 1} \quad \rightarrow \quad \text{AND} \quad \rightarrow \quad \text{Output} \]

\[ \text{Input 2} \]

3. Draw a picture that shows how to make a Not gate out of a Nor gate. (Hint: remember how a Not gate is made out of a Nand gate.)

\[ \text{Input} \quad \rightarrow \quad \text{NOT} \quad \rightarrow \quad \text{Output} \]
4. Now that you know how to make a Not gate out of a Nor gate, draw a picture that shows how to make an Or gate out of Nor gates.

\[ \text{Input 1} \quad \text{Input 2} \quad \text{Output} \]

5. Complete the table below, which completely describes the behavior of the following logic circuit:

\[ \text{Input 1} \quad \text{Input 2} \quad \text{Output} \]

<table>
<thead>
<tr>
<th>Input 1</th>
<th>Input 2</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>F</td>
<td>?</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>?</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>?</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>?</td>
</tr>
</tbody>
</table>

Compare this table with the table that describes the And gate. What do you notice? They tables are identical.
We have shown that a Not gate, an Or gate, and an And gate can be made out of \textit{Nor} gates.

**Conclusion:** the Nor gate is universal!

6. Recall that a Nand gate is equivalent to an And gate followed by a Not gate:

\[
\begin{array}{c}
\text{Input 1} \\
\text{Input 2}
\end{array}
\rightarrow
\neg
\rightarrow
\text{Output}
\]

How can you make a Nand gate out of Nor gates?
7 Application: Outdoor security light

On the front porch of a house, there is a motion detection device that outputs “True” if it detects that someone is in the front yard, and “False” otherwise.

There is also a solar sensor which outputs “True” if it detects that the sun is shining, and “False” otherwise.

And finally there is a master switch, which outputs “True” if it’s in the up position, and “False” otherwise.

An outdoor security light has a single input. The light turns on when the input has the value “True”, and otherwise the light is off.

Design a logic circuit that connects the security light to the motion detector, the solar sensor, and the master switch, so that:

- The light turns on when someone is in the yard and the sun is not shining, or the master switch is in the up position.
- Otherwise, the light is off.
8 Adding numbers with logic gates

We usually write numbers using “decimal notation”. In decimal notation, the number ten is written as 10.

However, it’s also possible to write numbers using “binary notation”. In binary notation, the number two is written as 10.

1. Complete the following addition problems. Write your answer using binary notation.

\[\begin{array}{c}
+ \ 0 \\
\hline
\ 0 \\
\end{array} \quad \begin{array}{c}
+ \ 0 \\
\hline
\ 1 \\
\end{array} \quad \begin{array}{c}
+ \ 1 \\
\hline
\ 1 \\
\end{array} \quad \begin{array}{c}
+ \ 1 \\
\hline
\ 10 \\
\end{array}\]

We will learn more about binary notation at another time. For now, this is all we need to know.
2. Complete the table below, which describes the behavior of this logic circuit.

But this time, we will use the number 0 as a short way of writing “False”, and we will use 1 as a short way of writing “True”.

<table>
<thead>
<tr>
<th>Input 1</th>
<th>Input 2</th>
<th>Output 1</th>
<th>Output 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>? 0</td>
<td>? 0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>? 0</td>
<td>? 1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>? 1</td>
<td>? 1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>? 1</td>
<td>? 0</td>
</tr>
</tbody>
</table>

Compare this table with the addition problems you did on the previous page. What do you notice? The output is the sum of the inputs (using binary notation).

This special logic circuit is the basic building block for more complicated logic circuits which are able to add large numbers.
9 Math Kangaroo questions

These questions are from the 2013 Canadian Math Kangaroo for 3rd and 4th graders.

1. Nancy bought 17 cones of ice-cream for her three children. Misha ate twice as many cones as Ana. Dan ate more ice-cream than Ana but less than Misha. How many cones of ice-cream did Dan eat?

   Dan: 5
   Ana: 4
   Misha: 8

2. Cristi has to sell 10 glass bells that vary in price: 1 dollar, 2 dollars, 3 dollars, 4 dollars, 5 dollars, 6 dollars, 7 dollars, 8 dollars, 9 dollars, 10 dollars. Is it possible to divide all the glass bells into three packages so that all the packages have the same price?

   No, $1+2+3+4+5+6+7+8+9+10 = 55$,
   and 55 is not divisible by 3.
3. In December Tom-the-cat slept for exactly 3 weeks. Which calculations should we do in order to find how many minutes he stayed awake during this month?

(a) \((31 - 7) \times 3 \times 24 \times 60\)
(b) \((31 - 7 \times 3) \times 24 \times 60\)
(c) \((30 - 7 \times 3) \times 24 \times 60\)
(d) \((31 - 7) \times 24 \times 60\)
(e) \((31 - 7 \times 3) \times 24 \times 60 \times 60\)

4. There are oranges, apricorns and peaches in a big basket. How many fruits are there in the basket if the peaches and the apricorns together are 18, the oranges and the apricorns together are 28, and 30 fruits are not apricorns?

\[
p + a = 18
\]
\[
\hat{o} + a = 28
\]
\[
p + \hat{o} = 30
\]

\[\Rightarrow 2p + 2\hat{o} + 2a = 76\]

\[\Rightarrow p + \hat{o} + a = \boxed{38}\]
5. In the shown triangle, first we join the midpoints of all the three sides. This way, we form a smaller triangle. We repeat this one more time with the smaller triangle, forming a new even smaller triangle, which we color in red. How many triangles of the size of the red triangle are needed to cover completely the original triangle, without overlapping?

6. Children in the school club had to arrange fitness balls according to their sizes from the biggest to the smallest one. Rebecca was comparing them and said: the red ball is smaller than the blue one, the yellow one is bigger than the green one, and the green one is bigger than the blue one. What is the correct order of the fitness balls?

\[ \text{red} < \text{blue} < \text{green} < \text{yellow} \]
7. In the Adventure Park, 20 children took part in two of the adventures. 15 of them participated in the “moving bridge” contest, and 20 of them went down the zip-wire. How many of the children took part in both adventures?

15

8. John is 33 years old. His three sons are 5, 6 and 10 years old. In how many years will the three sons together be as old as their father?

6 years