The Art of Word Problem Solving

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Today we're going to practice solving word problems.

Example: Joey goes over to his friend Dylan's house to play. He brings his coin collection. Joey and Dylan have a combined collection of 30 coins, but Dylan has four more coins than Joey. How many do each of the boys have?

Hint: Sometimes it helps to draw a picture!

Joey: \[\includegraphics[width=1cm]{joey_coins.png}\]
Dylan: \[\includegraphics[width=1cm]{dylan_coins.png}\]

The two boys have a common part which is represented by the long rectangle. The common part is the number of coins Joey owns, while for Dylan the common part plus 4 is equal to the number of coins he owns. This is shown by the four extra boxes. To find the common part, notice that if Dylan would give away his 4 extra coins, he and Joey would have the same number of coins.

\[30 - 4 = 26\]

\[26 \div 2 = 13\]

13 is the common part. Thus, Joey has 13 coins and Dylan has 13 + 4 = 17 coins.
2. Sarah and Mackenzie were comparing the number of books they had in their rooms. They found that if they put their collection together, they would have 242 books. However, Mackenzie realized that she had 10 less books than Sarah. How many books does Mackenzie have?

(a) Start by drawing a picture similar to the example.

Mackenzie:  
Sarah:  

(b) What is the common part? Show your work.

\[ 242 - 10 = 232 \]

\[ \frac{232}{z} = \boxed{116} \]

(c) How many books does Mackenzie have?

\[ \boxed{116} \]
4. Max eats up a jar of strawberry jam in 15 days, and Bella does the same in 10 days.

(a) What is the smallest number of days in which Max eats a whole number of jars and Bella eats a whole number of jars? (Hint: this number should be the smallest number which is divisible by 10 and by 15.)

\[ \text{Max: } 10 \text{ days} \]
\[ \text{Bella: } 15 \text{ days} \]

(b) How many jars of jam will each of them eat during this period?

\[ \text{Max: } -10 \text{ jars} \]
\[ \text{Bella: } -15 \text{ jars} \]

(c) How many jars of jam will they eat together in this time period?

\[ \text{Total: } 25 \text{ jars} \]

(d) How long does it take them to finish 1 jar of strawberry jam? (Hint: use your answer from part b.)

\[ \frac{10}{150} x + \frac{1}{10} x = 1 \]
\[ \frac{10}{150} x + \frac{15}{150} x = 1 \]
\[ \frac{25}{150} x = 1 \]
\[ x = \frac{150}{25} = 6 \text{ days} \]
6. Three brothers received the following inheritances from their father: 7 full chests of gold, 7 half-full chests of gold, and 7 empty chests. How can they divide this inheritance in such a way that each brother receives the same amount of gold and the same number of chests. (Transferring gold between chests is NOT allowed.)

(a) First, find out how many chests are there? How many chests should each brother get? (Not worrying about the amount of gold.)

\[ 7 + 7 + 7 = 21 \]

(b) How much gold are there all together? (Remember that 2 half chests make a whole chest). How much gold should each brother get?

\[ \frac{7 + \frac{7}{2}}{3} = 7 + 3.5 = 10.5 \text{ chests of gold} \]

\[ \frac{10.5}{3} = 3.5 \text{ chests of gold} \]

(c) Draw a picture.

\[ \frac{21}{3} = 7 \text{ each brother needs} \]

(d) How can the brothers successfully divide the inheritance?

Brother #1 = 3 full, 3 half, 5 empty
Brother #2 = 3 full, 3 half, 5 empty
Brother #3 = 1 full, 5 half, 1 empty